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Nonlinear convection in a non-Darcy porous medium*

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Abstract In this paper, the natural convection in a non-Darcy porous medium is studied using a temperature-concentration-dependent density relation. The effect of the two parameters responsible for the nonlinear convection is analyzed for different values of the inertial parameter, dispersion parameters, Rayleigh number, Lewis number, Soret number, and Dufour number. In the aiding buoyancy, the tangential velocity increases steeply with an increase in the nonlinear temperature parameter and the nonlinear concentration parameter when the inertial effect is zero. However, when the inertial effect is non-zero, the effect of the nonlinear temperature parameter and the nonlinear concentration parameter on the tangential velocity is marginal. The concentration distribution varies appreciably and spreads in different ranges for different values of the double dispersion parameters, the inertial effect parameter, and also for the parameters which control the nonlinear temperature and the nonlinear concentration. Heat and mass transfer varies extensively with an increase in the nonlinear temperature parameter and the nonlinear concentration parameter depending on Dacry and non-Darcy porous media. The variation in heat and mass transfer when all the effects, i.e., the inertial effect, double dispersion effects, and Soret and Dufour effects, are simultaneously zero and non-zero. The combined effects of the nonlinear temperature parameter, the nonlinear concentration parameter and buoyancy are analyzed. The effect of the nonlinear temperature parameter and the nonlinear concentration parameter and also the cross diffusion effects on heat and mass transfer are observed to be more in Darcy porous media compared with those in non-Darcy porous media. In the opposing buoyancy, the effect of the temperature parameter is to increase the heat and mass transfer rate, whereas that of the concentration parameter is to decrease.

Key words non-Darcy porous medium, natural convection, double dispersion

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Nomenclature

C,	dimensional concentration;
C_{s},	concentration susceptibility;
C_{p},	specific heat at constant pressure;
c,	inertial coefficient;
d,	pore diameter;
D,	constant molecular diffusivity;
D_X,	effective solutal diffusivity inthe X direction;
D_Y,	effective solutal diffusivity in the Y direction;
g,	acceleration due to gravity;
h,	heat transfer coefficient;
K,	permeability of the porous medium;
k,	effective thermal conductivity;

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k_{e}, thermal conductivity of the porous medium;
k_{T}, thermal diffusion ratio;
T, dimensional temperature;
U, dimensional velocity component along the X direction;
V, dimensional velocity component along the Y direction;
υ, velocity representation in three dimensions;
q, local surface heat flux;
$\frac{\partial p}{\partial X}$, pressure gradient in the flow direction.

Greek symbols

α, constant thermal diffusivity;
α_{d}, dynamic viscosity;
α_X, effective thermal diffusivity in the X direction;
α_Y, effective thermal diffusivity in the Y direction;
β_0, β_1, thermal expansion coefficients;
β_2, β_3, solutal expansion coefficients;
γ, thermal dispersion coefficient;
ξ, solutal dispersion coefficient;
ν, fluid kinematic viscosity;
ρ, fluid density;
μ, fluid viscosity;
η, similarity variable;
ψ, dimensional stream function;
θ, non-dimensional temperature;
ϕ, non-dimensional concentration;
θ_{w}, $\theta_{\mathrm{w}} = T_{\mathrm{w}} - T_\infty$;
ψ_{w}, $\phi_{\mathrm{w}} = C_{\mathrm{w}} - C_\infty$.

Parameters

$Le = \frac{\alpha}{D}$, diffusivity ratio (Lewis number);
$\alpha_1 = \frac{\beta_0}{2\beta_1(T_{\mathrm{w}} - T_\infty)}$, nonlinear temperature parameter;
$\alpha_2 = \frac{\beta_2}{2\beta_3(C_{\mathrm{w}} - C_\infty)}$, nonlinear concentration parameter;
$F = \frac{2c\alpha\sqrt{K}}{\nu X}$, inertial effect parameter;
$Ra_d = \frac{Kg\beta_0\theta_{\mathrm{w}}d}{\alpha\nu}$, pore dependent Rayleigh number;
$Ra_X = \frac{Kg\beta_0\theta_{\mathrm{w}}X}{\alpha\nu}$, local Rayleigh number;
$Ra_\xi = \xi Ra_d$, solutal dispersion parameter;
$Ra_\gamma = \gamma Ra_d$, thermal dispersion parameter;
$N = \frac{\beta_2\phi_{\mathrm{w}}}{\beta_0\theta_{\mathrm{w}}}$, buoyancy parameter;
Re_{p}, Reynolds number based on a typical pore or particle diameter;
D_{f}, Dufour effect parameter;
Sr, Soret effect parameter.

Subscripts

w, on the wall;
∞, at the outer edge of the boundary layer.

1 Introduction

The thermal and solutal transport by fluid flowing through a porous matrix is a phenomenon of great interest for both theories and applications. The studies of heat transfer in fluid saturated porous media can give insight into the understanding of the dynamics of hot underground springs, terrestrial heat flow through aquifier, and heat exchanges with fluidized beds. The studies of mass transfer can be applied in the miscible displacements in oil reservoirs, the spreading of solutes in fluidized beds, and the salt leaching in soils.

Coupled heat and mass transfer in porous media is gaining attention due to its interesting applications. The relationship between the two buoyancy effects driving the flow, namely, the density difference caused by temperature variations and the density difference caused by concentration variations, has many important applications including pore water convection near salt domes, doping processes, and cooling of electric equipments. In practice, the concentration differences are created by coating the surface with evaporating material which evaporates due to the heat of the surface or by injecting foreign gases like carbon dioxide, hydrogen, oxygen, helium, ammonia, and water vapor, etc. The combined heat and mass transfer in free convection under boundary layer approximations has been studied by Lai and Kulacki[1] and Angirasa et al.[2].

Mixing and recirculation of local fluid streams occur when the fluid moves through tortuous paths in packed beds. When the inertial effects are prevalent, the thermal and solutal dispersion effects become important. These effects are significant in forced and mixed convection flows as well as in vigorous natural convection flows. Detailed discussions regarding the effect of double dispersion on convection are available in Refs. [3] and [4].

In industrial and chemical engineering processes involving multi-component fluid, concentrations resulting from mass transfer vary from point to point. Heated jets or diffusion flames created by blowing combustible gases from a vertical pipe are controlled by the forced convection in the initial region and by the buoyancy forces far from the jet or pipe. The simplest physical model of such flows is the two-dimensional laminar flow along a vertical flat plate. Recent applications of this model can be found in the area of reactor safety, combustion flames, and solar collectors. The influence of Soret and Dufour effects on the flow field in free convection boundary layers from a vertical surface embedded in a Darcian porous medium has been studied by Postelnicu[5] and Anghel et al.[6].

The basis of the Boussinesq approximation is that there are flows in which the temperature varies a little. Therefore, the density varies a little and the buoyancy drives the motion. Thus, the variation in density is neglected everywhere except in the buoyancy term. Assuming that densities vary linearly with temperature variations, Cheng and Minkowcyz[7], Cheng[8], and Bejan and Khair[9] made an analysis for the natural convection. They assumed that the convection took place in a very thin layer around the heating surface. However, when the temperature and concentration difference between the plate and the ambient fluid was appreciably large, e.g., the study of the heat transfer rate and the size of the hot water zone around a dike, the mathematical modeling using a linear density temperature relation becomes more inaccurate. For example, when discussing the buoyancy induced boundary layer flows in geothermal reservoirs, Cheng[10] reported that with a maximum temperature of 1 200 °C, the intrusives are most likely to be cooled.

There are several reasons for the density temperature relationship to become nonlinear. Thermal stratification and heat released by the viscous dissipation, e.g., wall jet like profiles, induce significant changes in density gradients. In practice, the used liquid metals have the Prandtl numbers ranging from 0.03 to 0.003. Densities of fluids with very small Prandtl numbers, e.g., liquid sodium, change significantly. Therefore, a large boundary layer thickness is encountered in the free convection from a vertical plate.

It is obvious that when the temperature-concentration-dependent density relation is not accurate, the computation of the velocity of the fluid, non-dimensional heat and mass transfer coefficients are not accurate up to the desired degree of accuracy. Hence, the design of the thermal system will not be optimum. In fluidized bed technology, the fluid passes through a perforated or porous plate first, and then a bed of particles in a wide size distribution. To achieve the desired thermal performance, the velocity of the air passing through the bed of particles should be calculated exactly. Some of the thermal systems need cooling of transpiration, cooling of electronic components, turbine blades, etc., or drying of the surface, i.e, warm air blows along the surface to raise the local air temperature to evaporate water droplets and films. The designer of a cooling system has to provide thermal path ways so that the heat current can be discharged. The removal of heat by a coolant that sweeps or baths the warm surface is one of the most encountered cooling strategies in engineering systems. The layered porous media of high thermal conductivity architecture, e.g., metallic foams and sponges, for maximal cooling are distributed within the flow system. When the determination of non-dimensional heat and mass transfer coefficients is not accurate due to the inaccurate temperature-concentration-dependent density relation, the transpiration process or drying of the surface is simply continued. This will increase the rate of clogging and thermophoretical deposition rates[11]. In this process, all kinds of contaminants presented in the fluid, such as lint, dust, moisture, and even oil, are deposited on the surface. It should be remembered that the dust settled on the electronic components acts as an insulation layer, which makes it very difficult for the heat generated in the components to escape. Verms[12] studied the deposition rates in cooled and uncooled turbine cascades. It was found that the temperature difference between the wall and the gas could cause a fifteen-fold increase in the deposition rate as compared with the case of adiabatic cascade.

Goren[13] established the necessity of using the quadratic-density-temperature variation, namely, $\Delta\rho = -\rho\gamma(T - T_s)^2$. Similar theoretical studies can be found in Refs. [14–16]. Yen[17] conducted an experimental study to investigate the effect of density inversion on free convective heat transfer in a porous layer heated from the glass beads in water constituted the porous medium. The motivation to use the temperature-concentration-dependent relation are as follows.

(i) Some of the thermal systems, e.g., areas of reactor safety, combustion, solar collectors, layered porous media of high thermal conductivity architecture (metallic foams and sponges), are operated at moderate and very high temperature and use liquid metals of very small Prandtl numbers. In such special cases, the temperature-concentration-dependent relation is nonlinear and the Soret, Dufour, and double dispersion effects are of immense importance.

(ii) So far, the nonlinear temperature-concentration-dependent density has not been used to study the combined effect of the Soret, Dufour, and double dispersion.

(iii) The consequences due to the usage of linear temperature-concentration-dependent density relation as mentioned above are very serious.

(iv) The applications related are of immense importance. It is relevant here to analyze the effect of the temperature-concentration-dependent density nonlinear relation on the convective transport in a non-Darcy porous medium.

2 Governing equations

Consider the problem of natural convection from a flat surface embedded in a fluid-saturated non-Darcy porous medium. The temperature difference and the concentration difference between the plate and the medium are assumed to be large. Hence, the convection region is thick. The X-axis is taken along the plate and the Y-axis is normal to it. It is assumed that the fluid and the porous medium have constant physical properties, the fluid flow is moderate, and the permeability of the medium is low so that the Forchheimer flow model is applicable. With the Boussinesq approximation, the governing equations for the boundary layer flow from the wall to the fluid-saturated porous medium can be written as

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{1}$$

$$\frac{\partial U}{\partial Y} + \frac{c\sqrt{K}}{\nu}\frac{\partial^2 U}{\partial Y^2} = \frac{Kg}{\nu}(\beta_0(T - T_\infty) + \beta_1(T - T_\infty)^2 + \beta_2(C - C_\infty) + \beta_3(C - C_\infty)^2), \tag{2}$$

$$U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{\partial}{\partial X}\Big(\alpha_X\frac{\partial T}{\partial X}\Big) + \frac{\partial}{\partial X}\Big(\alpha_Y\frac{\partial T}{\partial Y}\Big) + \frac{Dk_{\mathrm{T}}}{C_{\mathrm{s}}C_{\mathrm{p}}}\Big(\frac{\partial^2 C}{\partial Y^2} + \frac{\partial^2 C}{\partial X^2}\Big), \tag{3}$$

$$U\frac{\partial C}{\partial X} + V\frac{\partial C}{\partial Y} = \frac{\partial}{\partial X}\Big(\alpha_X\frac{\partial C}{\partial X}\Big) + \frac{\partial}{\partial Y}\Big(D_Y\frac{\partial C}{\partial Y}\Big) + \frac{Dk_{\mathrm{T}}}{C_{\mathrm{s}}C_{\mathrm{p}}}\Big(\frac{\partial^2 T}{\partial Y^2} + \frac{\partial^2 T}{\partial X^2}\Big) \tag{4}$$

along with the boundary conditions

$$Y = 0, \quad V = 0, \quad T = T_{\mathrm{w}}, \quad C = C_{\mathrm{w}}, \tag{5}$$

$$Y \to \infty, \quad U \to 0, \quad T \to T_\infty, \quad C \to C_\infty. \tag{6}$$

It is suggested that the effective thermal diffusivity in a saturated porous medium can be expressed as $\alpha_Y = \alpha + \alpha_{\mathrm{d}}$, where α is the constant thermal diffusivity and α_{d} is the dynamic diffusivity due to mechanical dispersion. It is assumed that $\alpha_{\mathrm{d}} = \gamma d U$[3], where d is the dimension of the particle or pore diameter, and γ is the dispersion coefficient, which is a function of the strucfure of the porous medium. The solutal dispersion diffusivity can be written as $D_Y = D + \xi dU$[3], where ξ is the solutal dispersion coefficient. The velocity components in terms of stream function ψ can be written as $U = \frac{\partial\psi}{\partial Y}$, $V = -\frac{\partial\psi}{\partial X}$. This representation is valid since the expressions for the velocity components clearly satisfy the continuity equation. Use the following similarity transformation:

$$\eta = \frac{Y}{X} Ra_X^{\frac{1}{2}}, \quad \psi = f(\eta)\alpha Ra_X^{\frac{1}{2}}, \quad \theta(\eta) = \frac{T - T_\infty}{T_\mathrm{w} - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_\mathrm{w} - C_\infty}.$$

A set of partial differential equations (1)–(4) is transformed into the following ordinary differential equations:

$$f'' Ra_X^{\frac{1}{2}} + F f' f'' Ra_X = \frac{1}{2}(\eta\theta'(1 + \pi_1\theta) + N\eta\phi'(1 + \pi_2\theta)), \tag{7}$$

$$\theta'' + \frac{1}{2} f\theta' + Ra_\gamma (f'\theta'' + f''\theta') + D_\mathrm{f}\phi'' = 0, \tag{8}$$

$$\phi'' + \frac{Le}{2} f\phi' + LeRa_\xi (f'\phi'' + f''\phi') + SrLe\theta'' = 0. \tag{9}$$

The boundary conditions are as follows:

$$\eta = 0 : F = 0, \quad \theta = 1, \quad \phi = 1, \tag{10}$$

$$\eta \to \infty : f' \to 0, \quad \theta \to 0, \quad \phi \to 0. \tag{11}$$

The non-dimensional heat and mass transfer coefficients are written as

$$\frac{Nu_X}{Ra_X^{\frac{1}{2}}} = -(1 + Ra_\gamma f'(0))\theta'(0), \tag{12}$$

$$\frac{Sh_X}{Ra_X^{\frac{1}{2}}} = -(1 + Ra_\xi f'(0))\phi'(0). \tag{13}$$

3 Results and discussions

The ordinary differential equations (7)–(9) along with the boundary conditions (10) and (11) are integrated by giving appropriate initial guess values for $f'(0)$, $\theta'(0)$, and $\phi'(0)$ to match the values with the corresponding boundary conditions $f'(\infty)$, $\theta'(\infty)$, and $\phi'(\infty)$, respectively. Nag software (DO2HAEF routine) is used for integrating the corresponding first-order system of equations and for shooting and matching the initial boundary conditions. The results obtained are accurate up to the fourth decimal place.

If a set of values $Ra_X{=}4$, $\eta{=}4$, and $\pi_1 = \pi_2{=}0$ are assigned to the system of ordinary differential equations (7)–(9), it will get reduce to the "Soret, Dufour effects in a non-Darcy porous medium for a specific magnetic field M = 0" [18]. Hence, by assigning $Ra_X{=}4$, $\eta{=}4$, $F{=}0$, $Ra_\gamma{=}0$, $Ra_\xi{=}0$, and $Le{=}3$, the non-dimensional heat and mass transfer coefficients are determined for various values of D_f and Sr, The results are presented in Table 1. For a comparison, the results of the non-dimensional heat and mass transfer with the same values of parameters, i.e., $M{=}0$, $F{=}0$, $Ra_\gamma{=}0$, $Ra_\xi{=}0$, and $Le{=}3$, and the same values of D_f and Sr in the governing equations obtained by Partha et al.[18] are also listed in Table 1. The same software, i.e., Nag software (DO2HAEF routine) is used for the governing equations. It is observed that the present results agree well with those of Ref. [18].

The change rate of the tangential velocity f' with the increase of the parameter α_2 is clearly shown in Fig. 1. The tangential velocity f' increases with the increase in α_2. However, the increase is different for the zero and non-zero inertial effects. When the inertial effect is zero, it

Table 1 Non-dimensional heat and mass transfer coefficients with $Ra_X = 4$, $\eta = 4$, $F = Ra_\gamma = Ra_\xi = 0$, $Le = 3$, and $N = 1$ except that $\alpha_1 = \alpha_2 = 0$ in the present results and the magnetic field $M = 0$ in Ref. [18]

Parameters	$\frac{Nu_X}{Ra_X^{1/2}}$		$\frac{Sh_X}{Ra_X^{1/2}}$	
	Pres.	Ref. [18]	Pres.	Ref. [18]
$D_f = 0.4$, $Sr = 0.6$	1.345 751	1.345 751	0.267 343	0.267 343
$D_f = 0.4$, $Sr = 0.4$	0.902 138	0.902 138	0.577 477	0.577 477
$D_f = 0.2$, $Sr = 0.4$	0.717 100	0.717 100	0.746 955	0.746 955

increases steeply with the increase of α_2. When it is non-zero, i.e., $Ra_\gamma=0$, $F=5$, $Ra_\xi=0$ or $Ra_\gamma=5$, $F=5$, $Ra_\xi=5$, the effect of α_2 on f' is marginal. The effect of α_1 on f' is the same as that of α_2. Thus, we conclude that the significant effect of α_1 or α_2 on f' depends on the magnitude of the inertial parameter and dispersion parameters.

The temperature distributions with the variation of η for different values of α_1 and α_2 are well presented in Fig. 2. It is understandable from the figure that the thermal boundary layer thickness decreases as the parameter values of α_1 or α_2 increase. A careful observation shows that the effect of α_1 on the temperature distribution seems to be more prominent compared with that of α_2. In another words, when α_1 and α_2 increase from 0.1 to 5.0, the thermal boundary layer thickness corresponding to α_1 reduces drastically.

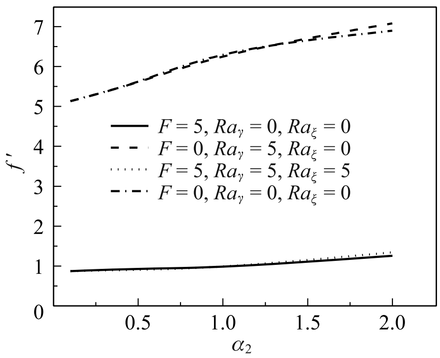

Fig. 1 Velocity distribution inside the boundary layer via α_2 with $Ra_X = 5$, $Le = 10$, $N = 1$, $\alpha_1 = 1$, and $D_f = Sr = 0$

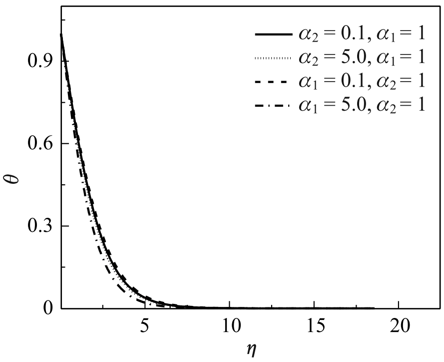

Fig. 2 Temperature distribution inside the boundary layer via η with $Ra_\gamma = 0$, $Ra_\xi = 0$, $F = 5$, $D_f = Sr = 0$, $Ra_X = 5$, $Le = 10$, and $N = 1$

The dependence of α_1 or α_2 on the inertial parameter and double dispersion parameters affecting the concentration distribution can be analyzed from Fig. 3 and Fig. 4. Though the concentration boundary layer thickness decreases with the increase of α_1 or α_2, irrespective of the values of the inertial parameter and double dispersion parameters, the concentration distribution varies appreciably depending on the combination of several parameters. The concentration distribution corresponding to $Ra_\gamma = 5$, $F = 0$, and $Ra_\xi = 0$ is spread in the range of $0 \leqslant \eta \leqslant 0.8$. When $Ra_\xi = 5$, $F = 0$, and $Ra_\gamma = 0$, it is $0 \leqslant \eta \leqslant 4.0$. Here also, the effect of α_1 on the concentration boundary layer thickness appears to be more compared with the effect of α_2 (Fig. 3). Hence, it is apparent that the parameters α_1 and α_2 influence the temperature field and the concentration field to a greater extent.

Figures 5–7 depict that the influence of the nonlinear parameters on heat and mass transfer varies in several ways. It depends not only on the dispersion and inertial parameters but also on the Lewis number. The common feature observed is that the non-dimensional heat and mass

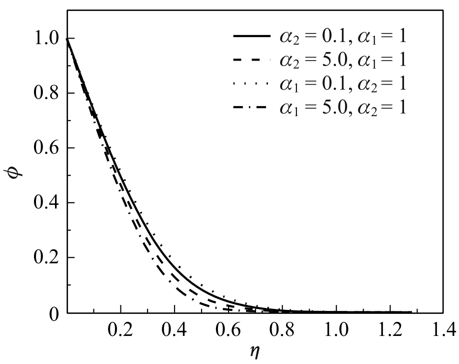

Fig. 3 Concentration distribution inside the boundary layer via η with $Ra_\gamma = 5$, $Ra_\xi = 0$, $F = 0$, $D_\mathrm{f} = Sr = 0$, $Ra_X = 5$, $Le = 10$, and $N = 1$

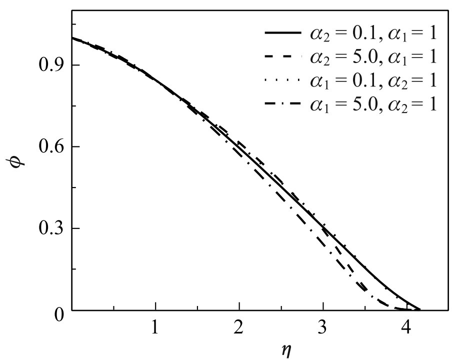

Fig. 4 Concentration distribution inside the boundary layer via η with $Ra_\gamma = 0$, $Ra_\xi = 5$, $F = 0$, $Ra_X = 5$, $Le = 10$, and $N = 1$

transfer coefficients increase with the increase of α_1 or α_2 up to a certain stage, and the change of this trend depends on the magnitude of several parameters. From Fig. 5, we can observe that the effect of α_2 on the heat transfer up to $\alpha_2 = 1.0$ is considerable, and later the effect becomes marginal. From Fig. 5, we can also see that the heat transfer increases continuously with α_1 or α_2. As shown in Fig. 6, when $F = 5$, $Ra_\gamma = 5$, and $Ra_\xi = 5$, heat transfer increases almost at the same level with the increase of α_1 and α_2; but when $F = 0$, $Ra_\gamma = 0$, and $Ra_\xi = 0$, heat transfer increases at different levels with the increase of α_1 and α_2, and the increase with α_2 is marginal. It is always very essential to estimate the magnitude of heat transfer since the quality of the final product depends on the heat transfer from the surface. In this context, it is worth mentioning that, for fixed values of F, Ra_γ, Ra_ξ, and Le, with the increase of α_1 or α_2, the non-dimensional heat transfer coefficient varies between 2 to 4.5 and 0.2 to 1.12 as shown in Fig. 5 and Fig. 6, respectively. From Fig. 7, It is seen that mass transfer increases with α_2 up to a certain stage first, and thereafter tends to decrease with the increases of α_2. Figure 7 clearly shows that mass transfer variation depends not only on the dispersion parameters but also on the nonlinear parameters. It is quite interesting to note that the effect of α_1 and α_2 is more on heat transfer (Fig. 7, not enclosed) and very less on mass transfer when the parameters controlling the inertial effect and the double dispersion effects are non-zero. When the parameters are zero, the effect of α_1 and α_2 on mass transfer is moderate, whereas heat transfer increases with the increase of α_1 and is slightly affected with the increase of α_2.

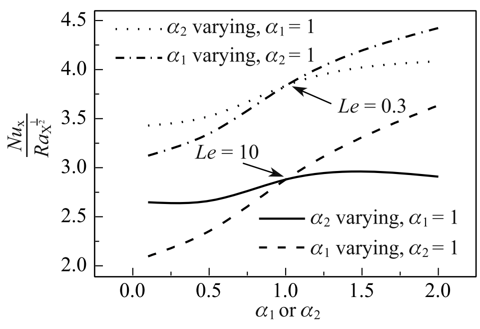

Fig. 5 Variation of non-dimensional heat transfer coefficient via α_1 or α_2 with $Ra_\gamma = 5$, $Ra_\xi = 0$, $F = 0$, $D_\mathrm{f} = Sr = 0$, $Ra_X = 5$, and $N = 1$

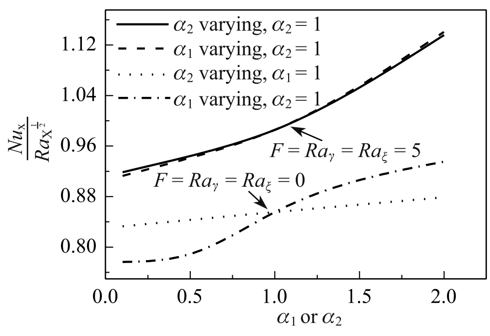

Fig. 6 Variation of non-dimensional heat transfer coefficient via α_1 or α_2 with $D_\mathrm{f} = Sr = 0$, $Ra_X = 5$, $Le = 10$, and $N = 1$

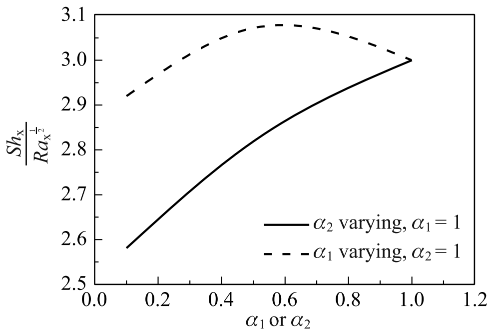

Fig. 7 Variation of non-dimensional mass transfer coefficient via α_1 or α_2 with $Ra_\gamma = 0$, $Ra_\xi = 5$, $F = 0$, $D_{\mathrm{f}} = Sr = 0$, $Ra_X = 5$, $Le = 10$, and $N = 1$

To establish the direct impact of the nonlinear parameters on buoyancy which in turn influences the convective transport, the buoyancy parameter is assigned variously and the analysis is done for ($\alpha_1 = 0$, $\alpha_2 = 1$), ($\alpha_1 = 1$, $\alpha_2 = 1$), and ($\alpha_1 = 1$, $\alpha_2 = 0$). It is observed that the tangential velocity, non-dimensional heat and mass transfer coefficients increase with the increase of the buoyancy parameter. Also, the rate of change is observed to be significant only when ($Ra_\gamma = 5$, $Ra_\xi = 0$, $F = 0$) or ($Ra_\gamma = 0$, $Ra_\xi = 5$, $F = 0$). Such an extent is not seen when $Ra_\gamma = 0$, $Ra_\xi = 0$, and $F = 5$.

The numerical values are generated for fixed values of Ra_γ, Ra_ξ, Le, F, D_{f}, and Sr and various valuses of $\alpha_1 = 0$, $\alpha_2 = 1$ and $\alpha_1 = 1$, $\alpha_2 = 0$ as listed in Table 2. It is very clear that the mass transfer changes significantly while heat transfer changes moderately with the increase of the buoyancy parameter. The results of various α_1 and α_2 clearly justify that the effect of buoyancy on heat and mass transfer depends on α_1 and α_2.

Table 2 Effects of aiding buoyancy on f' and non-dimensional heat and mass transfer coefficients with $F = Ra_\gamma = 0$ and $Ra_\xi = 5$ for case 1, i.e., $\alpha_1 = 0$, $\alpha_2 = 1$, and case 2, i.e., $\alpha_1 = 1$, $\alpha_2 = 0$

N	f'		$\frac{Nu_X}{Ra_X^{1/2}}$		$\frac{Sh_X}{Ra_X^{1/2}}$	
	Case 1	Case 2	Case 1	Case 2	Case 1	Case 2
1	5.031 15	5.031 15	1.137 912	1.073 967	3.018 3	2.522 1
2	8.049 84	7.043 61	1.489 506	1.331 558	5.277 5	4.106 7
4	14.087 2	11.068 5	2.021 511	1.737 846	9.909 1	7.388 0

The effect of the parameters α_1 and α_2 at various values of Le are shown in Table 3. Tables 2 and 3 depict that the marginal changes in heat and mass transfer, which is due to the Soret and Dufour effect when the parameters of the inertial dispersion effects are no-zero. When the parameters of the inertial, double dispersion and Soret, Dufour effects are zero with non-zero α_1 and α_2, the tangential velocity and mass transfer increase substantially, whereas the heat transfer decreases significantly. If all the above mentioned parameters are zero, the tangential velocity as well as heat and mass transfer decrease appreciably. When α_1 and α_2 increase, there will be enormous increase in the tangential velocity and mass transfer with considerable increase in heat transfer. From Table 3, it can be seen that for a higher value of $Le = 1\,000$, when α_1 and α_2 increase, the tangential velocity and mass transfer will considerably increase, whereas the heat transfer increases marginally. However, when α_1 and α_2 increase in Darcy porous media for the same value of $Le = 1\,000$, there will be significant increase in the tangential velocity as well as in the mass transfer, whereas the increase heat transfer is substantial. As shown in Table 3, when Le is assumed to be very small, i.e., $F = Ra_\gamma = Ra_\xi = 0$, $D_{\mathrm{f}} = 0.1$, $Sr = 0.6$, and $Le = 3$, there will be an extensive increase in the tangential velocity and a considerable increase in heat and mass transfer with the increase of α_1 and α_2. When the parameter Sr increases, heat and mass transfer changes extensively. Also, with the increase of the parameter $D_{\rm f}$, heat transfer increases while mass transfer decreases. When both the parameters $D_{\rm f}$ and Sr decrease, heat transfer decreases drastically while mass transfer increases considerably.

Table 3 Effects of various parameters on f' and non-dimensional heat and mass transfer coefficients

Parameters		f'	$\frac{Nu_X}{Ra_X^{1/2}}$	$\frac{Sh_X}{Ra_X^{1/2}}$
Same	Different			
$Le = 100,\ N = 1,\ Ra_X = 3$	$\alpha_1 = 0.2,\ \alpha_2 = 1,$ $Ra_\gamma = 5,\ Ra_\xi = 5,$ $D_{\rm f} = 0.1,\ F = 0.6,\ Sr = 0.5$	2.769 398	2.597 839	2.681 856
$F = Ra_\gamma = Ra_\xi = D_{\rm f} = Sr = 0$	$\alpha_1 = 0.2,\ \alpha_2 = 1$	6.754 997	0.791 308	12.627 699
	$\alpha_1 = 0,\ \alpha_2 = 0$	5.196 152	0.757 646	11.507 245
	$\alpha_1 = 1,\ \alpha_2 = 3$	10.392 303	0.896 450	15.137 700
$Le = 1\,000,$ $\alpha_2 = Ra_\xi = D_{\rm f} = Sr = 0$	$\alpha_1 = 0.1,\ F = 0.5,\ Ra_\gamma = 5$	1.125 474	0.716 982	17.564 664
	$\alpha_1 = 0.4,\ F = 0.5,\ Ra_\gamma = 5$	1.184 033	0.769 515	18.129 595
	$\alpha_1 = 0.4,\ F = Ra_\gamma = 0$	1.967 740	0.456 615	22.758 496
$\alpha_2 = 0.7,\ Le = 3,$ $F = Ra_\gamma = Ra_\xi = 0$	$\alpha_1 = 0.5,\ D_{\rm f} = 0.1,\ Sr = 0.6$	6.754 990	1.254 546	1.035 789
	$\alpha_1 = 0.7,\ D_{\rm f} = 0.2,\ Sr = 1.5$	7.014 806	5.512 490	13.331 984
	$\alpha_1 = 0.7,\ D_{\rm f} = 0.4,\ Sr = 0.6$	7.014 806	2.487 528	0.532 440

To sum up, the effect of α_1, α_2, $D_{\rm f}$, and Sr in Darcy porous media appears to be more compared with that in non-Darcy porous media, and the rate of change of the tangential velocity and heat and mass transfer depends on the magnitude of other related parameters. In the opposing buoyancy (tables not enclosed), a common characteristic observed is that increasing α_2 decreases the tangential velocity, heat transfer, and mass transfer irrespective of the values of Ra_γ, Ra_ξ, and F. But increasing α_1 results in the increase of the tangential velocity, heat transfer, and mass transfer. However, a considerable rate of increase or decrease depends on the complex interaction among several parameters.

4 Conclusions

A similarity solution technique is used to study the natural convection in a non-Darcy porous medium using the temperature-concentration-dependent density relation. The effect of the parameters α_1 and α_2 on convective heat and mass transfer are analyzed in Darcy and non-Darcy porous media for different values of buoyancy parameter, Rayleigh number, and Lewis number.

In the case of aiding buoyancy, the following conclusions can be drawn. The tangential velocity f' increases with the increase of the parameter α_1 or α_2, and it increases steeply in Darcy porous media and marginal in non-Darcy porous media. With the increase of α_1 and α_2, the thermal boundary layer thickness and the solutal boundary layer thickness decrease noticeably. The effects of α_1 and α_2 on the temperature distribution and the concentration distribution vary significantly according to the different values of α_1, α_2, F, Ra_γ, and Ra_ξ.

The non-dimensional heat and mass transfer coefficients increase with the increase of α_1 and α_2 in Darcy as well as non-Darcy porous media up to a certain stage for all values of Le. However, this trend may change. It depends on the magnitudes of several parameters. The effect of α_1 and α_2 is more on heat transfer and very less on mass transfer when the parameters controlling the inertial effect and double dispersion effects are non-zero. When these parameters are zero, the effect of α_1 and α_2 on mass transfer is moderate. Also, heat transfer increases with the increase of α_1 and α_2. It is slightly effected with the increase of α_2.

It is found that increasing the buoyancy parameter increases the tangential velocity and heat and mass transfer, which depends on the values of α_1 and α_2. This increase rate is significant in the absence of inertial and marginal parameters. Hence, the effect of buoyancy on heat and mass transfer depends not only on the inertial parameter and dispersion parameters, but also on α_1 and α_2.

Heat and mass transfer varies extensively with the increase of α_1 and α_2, which depends on Dacry and non-Darcy porous media. The effect of α_1, α_2, D_f, and Sr in Darcy porous media appears to be more compared with that in non-Darcy porous media. The change rates of the tangential velocity and the heat and mass transfer depends on the magnitudes of other related parameters. In the case of opposing buoyancy, the effect of α_1 is to increase the heat and mass transfer rate whereas that of α_2 is to decrease.

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