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COMPUTER SIMULATION OF THE MOTION OF THE BULLET BELT OF AIRPLANE GUN

Zhang Dingguo (章定国)[1]

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Abstract

With the dynamic theory of multi-rigidbody systems, first this paper establishes the mathematical model of dynamics and impact dynamics of the bullet belt of airplane gun, and then it carries out the numerical and graphic simulation of the motion of the bullet belt by way of computer.

Key words dynamics, bullet belt, collision, computer simulation

I. Introduction

Accompanying the development of the dynamic theory of multi-body systems, of the computer technique, and the demands of the high accuracy and the lightening for arms, people begin to pay attention to introducing dynamic simulation technique to the designing of arms. The traditional method, to improve and verify the function of arms only by way of "shooting", increasingly shows its malpractice. People have already found that simulation is playing a more and more important role in the designing of arms. By combining the dynamic theory with the computer technique, this paper studies the designing and optimization of the system of the bullet belt of one airplane gun for the first time, and develops a simulation software.

The phenomenon of "bullet block" occurring in the shooting procedure is a huge hidden danger. The world of engineering is anxious for research man to solve it. Expensive experiments cannot achieve the desired results. However, the method to combine dynamic theory with computer technique is a good method for us to demonstrate the actual shooting procedure again and to optimize the motion of the bullet belt.

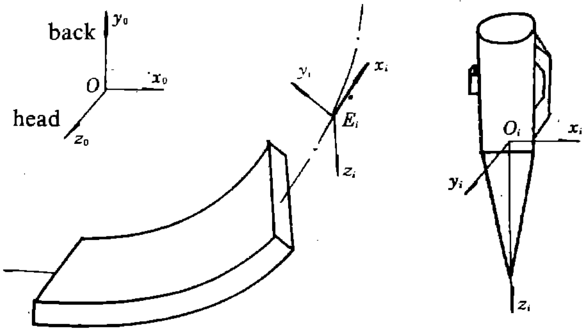

Fig. 1 The system of bullet belt

[1] Nanjing University of Science and Technology, Nanjing 210094, P. R. China

The actual system of the bullet belt of airplane gun can be abstracted into the bullet belt and the track, see Fig. 1. The belt consists of bullet links, and one bullet link is connected with another by hook and loop. There exists small space between hook and loop, and so the hook has six degrees of freedom relative to the loop. Because the bullet belt is restrained in the flat track, so it looses the translation along the y_t axis and the rotation revolving around the x_t axis.

II. Impact Dynamics in the System of Bullet Belt

2.1 Mathematical description of the impact problem

In the system of airplane gun bullet belt, there exist two kinds of impact problems of multi-rigidbody systems: the first is one with collision between bodies belonging to different systems, the second is one with collision between bodies belonging to just the same system. For the airplane gun bullet belt, the one with collision between the bullet belt and the track is the first, the one with collision between bullet bodies belonging to the belt is the second. Perhaps there exist several forms of collision as follows:

A. Collision between two or more bullet links;

B. None collision between two bullet links.

At the same time, s indicates the s-th collision segment, which is the number of times of form A of collision. The number of links in every segment is called length, and noted with n_s. In the form A of collision, there maybe exists collision between the bullet bodies and the track, so the form A of collision includes all two kinds of impact problems. There does not exist collision between bodies in form B of collision. It is singular bullet link. But maybe the singular bullet link collides with the track. So form B only occurs the first kind of collision problems. It is a degenerate situation of form A. From the viewpoint of mechanics, situation A is a kind of separate pattern treelike configuration multi-point collision which occurs between restrained bodies. At present, this kind of impact situation is discussed still seldom.

In the s-th collision segment, we renumber the bullet bodies from 1 to n_s. In order to consider the two kinds of impact problems in the meantime, we regard the track as "pseudobulletbody". and number it with n_{s+1}. And suppose the "extended bullet chain impact segment" has Q_s collision points. See Fig. 2. Full lines indicate the colliding body pairs. Of the two bullet bodies which consist of the k-th collision pair, the one with the smaller number is noted with index $l^+(k)$ and defined as inner bullet body, and the other with the larger number is noted with index $l^-(k)$ and defined as outer bullet body. The names of the bullet bodies are $B_{l^+(k)}$ and $B_{l^-(k)}$. The colliding body pairs k are numbered according to these rules: the smaller the number of the inner body, the smaller the number of the colliding pair is; if the numbers of the inner body are the same, then the smaller the number of the outer body, the smaller the number of the colliding pair is.

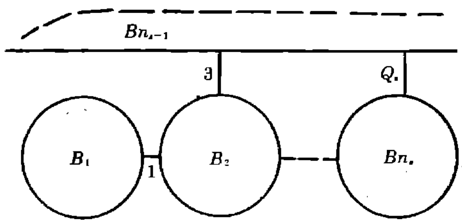

Fig. 2 Impact topology

Defining the collision incidence matrix W_s to describe the collision relation between bodies in the s-th segment:

$$W_s(l)(k)=\begin{cases} -1 & \text{when } l=l^+(k),\ l=1,2,\cdots,n_s+1 \\ +1 & \text{when } l=l^-(k) \quad k=1,2,\cdots,Q_s \\ 0 & \text{otherwise} \end{cases} \tag{2.1}$$

Defining $\boldsymbol{\rho}^*_{lk}$ are vectors from the l-th bullet body's center of mass to the k-th collision point. When $l\neq l^{\pm}(k)$, $\boldsymbol{\rho}^*_{lk}=0$.

Defining

$$\boldsymbol{\rho}_{lk}=W_s(l)(k)\cdot\boldsymbol{\rho}^*_{lk} \tag{2.2}$$

then $\boldsymbol{\rho}_{lk}$ consists of a $(n_s+1)\times Q_s$ vector matrix $\boldsymbol{\rho}$.

For generality, suppose bullet body l may collide with several others at the same time, the impulses acting on body l can be combined in a resultant impulse $\boldsymbol{I}_l$ with its line of action passing through the body's center of mass and a resultant external impulse torque $\boldsymbol{T}_l$. The relation between colliding point impulse $\boldsymbol{I}^*_l$ and $\boldsymbol{I}_l$, $\boldsymbol{T}_l$ can be expressed

$$\boldsymbol{I}=W_s\boldsymbol{I}^* \tag{2.3}$$

$$\boldsymbol{T}=\boldsymbol{\rho}\times\boldsymbol{I}^* \tag{2.4}$$

in which

$$\boldsymbol{I}^*=\boldsymbol{n}I^* \tag{2.5}$$

$$I^*=(I^*_1,I^*_2,\cdots,I^*_{Q_S})^T \tag{2.6}$$

$$\boldsymbol{n}=\mathrm{diag}(\boldsymbol{n}_1,\boldsymbol{n}_2,\cdots,\boldsymbol{n}_{Q_S}) \tag{2.7}$$

$\boldsymbol{n}_k$ is the unit vector normal to the tangent plane at the k-th collision point, its positive direction accords with the outer normal line of $B_{l^+(k)}$.

Unit vector $\boldsymbol{p}_l$ is defined as the direction of linear displacement restraint acting on B_l and $\boldsymbol{r}_l$ is defined as the direction of angular displacement restraint of B_l. The restraint impulse matrix and the restraint impulse torque matrix are

$$I^c=(I^c_1,I^c_2,\cdots,I^c_{n_s+1})^T \tag{2.8}$$

$$T^c=(T^c_1,T^c_2,\cdots,T^c_{n_s+1})^T \tag{2.9}$$

then

$$\boldsymbol{I}^c=\boldsymbol{p}I^c,\quad \boldsymbol{T}^c=\boldsymbol{r}T^c \tag{2.10)-(2.11}$$

where

$$\boldsymbol{p}=\mathrm{diag}(\boldsymbol{p}_1,\boldsymbol{p}_2,\cdots,\boldsymbol{p}_{n_s+1}) \tag{2.12}$$

$$\boldsymbol{r}=\mathrm{diag}(\boldsymbol{r}_1,\boldsymbol{r}_2,\cdots,\boldsymbol{r}_{n_s+1}) \tag{2.13}$$

2.2 Impact dynamics of the system of the bullet belt of airplane gun

Defining $\Delta\boldsymbol{v}^c$ is the vector matrix of velocity increment of center of mass and $\Delta\boldsymbol{\omega}$ is the vector matrix of angular velocity increment:

$$\Delta\boldsymbol{v}^c=(\Delta\boldsymbol{v}^c_1,\Delta\boldsymbol{v}^c_2,\cdots,\Delta\boldsymbol{v}^c_{n_s+1})^T \tag{2.14}$$

$$\Delta\boldsymbol{\omega}=(\Delta\boldsymbol{\omega}_1,\Delta\boldsymbol{\omega}_2,\cdots,\Delta\boldsymbol{\omega}_{n_s+1})^T \tag{2.15}$$

letting

$$U=(\Delta V_{l^-(1)}-\Delta V_{l^+(1)},\Delta V_{l^-(2)}-\Delta V_{l^+(2)},\cdots,\Delta V_{l^-Q_s)}-\Delta V_{l^+,Q_s})^T \qquad (2.16)$$

where $\Delta V_k(k=1,2,\cdots,Q_s)$, is the velocity increment of the k-th colliding point

According to kinematic theory, we can obtain

$$U=W_s^T\Delta v^c+\rho^T\times\Delta\omega \qquad (2.17)$$

and again with Newton-Euler impact theorem, obtaining

$$M\Delta v^c=I+I^c,\quad J\Delta\omega=T+T^c \qquad (2.18)-(2.19)$$

where

$$M=\mathrm{diag}(M_1,M_2,\cdots,M_{n_s+1}) \qquad (2.20)$$

$$J=\mathrm{diag}(J_1,J_2,\cdots,J_{n_s+1}) \qquad (2.21)$$

M_l,J_l are the mass and the pseudo inertia matrix of the l-th bullet body. Because "pseudobulletbody" is fixed on the airplane, so

$$M_{n_s+1}\gg M_l,\quad l=1,2,\cdots,n_s$$

In the course of collision, we have restraint equations:

$$p\cdot\Delta v^c=0,\quad r\cdot\Delta\omega=0 \qquad (2.22)-(2.23)$$

Supposing the Newton impact law is right or approximately right in this impact model, then there exist equations relating to the coefficient of restitution as follows

$$n\cdot u=-(\sigma+E)n\cdot Dv \qquad (2.24)$$

in which

$$DV=(DV_1,DV_2,\cdots,DV_{Q_s})^T \qquad (2.25)$$

$$DV_k=V_{l^-(k)}-V_{l^+(k)} \qquad (2.26)$$

$E=\mathrm{diag}(e_1,e_2,\cdots,e_{Q_s})$, where e_k is the coefficient of restitution at the k-th collision point σ is a $Q_s\times Q_s$ unit matrix. Eqs. (2.18), (2.19), (2.22), (2.23), (2.24) are the impact dynamic equations which we want to obtain.

Solving above equations, we obtain:

$$M\Delta v^c=(W_sn+p\alpha)I^*,\quad J\Delta\omega=(p\times n+r\beta)I^* \qquad (2.27)-(2.28)$$

where

$$\alpha=-p\cdot W_s\cdot n\ ,\quad \text{it is a } (n_s+1)\times Q_s \text{ matrix} \qquad (2.29)$$

$$\beta=-r\cdot(\rho\times n)\ ,\ \text{it is also a } (n_s+1)\times Q_s \text{ matrix} \qquad (2.30)$$

from Eqs. (2.27) and (2.28), Δv^c and $\Delta\omega$ are obtained, and substituting them to (2.17), we can obtain

$$U=HI^*\ ,\ \text{where } H \text{ is known} \qquad (2.31)$$

substituting Eq. (2.31) to Eq. (2.24), we obtain

$$n\cdot HI^*=-(\sigma+E)n\cdot DV \qquad (2.32)$$

In Eq. (2.32) only I^* is unknown, from (2.32) we obtain I^*, then we obtain $\Delta v^c,\Delta\omega,I^c,T^c$.

III. Non-Impact Dynamics of the Bullet Belt of Airplane Gun

At the time of non-impact, when the bullet belt moves along the track, all kinds of normal forces will affect the motion of the bullet belt. So all normal forces must be considered. Moreover, the flying situation of the airplane also affects the motion of the bullet belt.

Constructing the inertial coordinate frame $\boldsymbol{e}$, and letting $\boldsymbol{e}_3 /\!/ \boldsymbol{e}_3^{(0)}$. The velocity of the origin of the plane coordinate frame $\boldsymbol{e}^{(0)}$ is $\boldsymbol{v}_0$, and the tangential acceleration of the origin of $\boldsymbol{e}^{(0)}$ is $\boldsymbol{a}_0$. Ω is the angular velocity with which the plane makes a turn. $v_j^{(0)}$ or $\omega_j'^{(0)}$, $j=1, 2, 3$, is the projection on $\boldsymbol{e}^{(0)}$ about the velocity or the angular velocity of the center of mass of bullet body relative to the plane coordinate frame $\boldsymbol{e}^{(0)}$. Suppose airplane body turns round levelly with right slanting angle θ and radius R. According to the Newton kinematic equation of center of mass, we can obtain

$$m\boldsymbol{a}_r = \boldsymbol{N} - mg\boldsymbol{e}_2 + \boldsymbol{F}_n + \boldsymbol{F}_\tau + \boldsymbol{F}_K \tag{3.1}$$

where $\boldsymbol{e}_2$ is the 2nd axis of the inertial coordinate frame $\boldsymbol{e}$, and its direction is upward. $\boldsymbol{F}_n, \boldsymbol{F}_\tau, \boldsymbol{F}_k$ are the inertial centrifugal force, the tangential force and the Coriolis force. The vector equation, Eq. (3.1), includes three unknowns N, a_{r_1} and a_{r_3}. By projecting Eq. (3.1) to $\boldsymbol{e}^{(0)}$, we can obtain three scalar equations. So N, a_{r_1}, a_{r_3} can be obtained.

According to the theory of dynamics of rigidbody, the Euler dynamic equations expressed in spindle coordinate frame can be written as follows:

$$\left.\begin{aligned} J_{K1}\dot{\omega}_1^{(k)} + (J_{K3} - J_{K2})\omega_2^{(k)}\omega_3^{(k)} &= M_1^{(k)} \\ J_{K2}\dot{\omega}_2^{(k)} + (J_{K1} - J_{K3})\omega_3^{(k)}\omega_1^{(k)} &= M_2^{(k)} \\ J_{K3}\dot{\omega}_3^{(k)} + (J_{K2} - J_{K1})\omega_1^{(k)}\omega_2^{(k)} &= M_3^{(k)} \end{aligned}\right\} \tag{3.2}$$

where

$$\begin{pmatrix} M_1^{(k)} \\ M_2^{(k)} \\ M_3^{(k)} \end{pmatrix} = A^{k0} \begin{pmatrix} M\cos\varphi_2 \\ 0 \\ M\sin\varphi_2 \end{pmatrix} \tag{3.3}$$

$$\begin{pmatrix} \omega_1^{(k)} \\ \omega_2^{(k)} \\ \omega_3^{(k)} \end{pmatrix} = A^{k0} \begin{pmatrix} \omega_1'^{(0)} - \Omega\sin\theta \\ \omega_2'^{(0)} - \Omega\cos\theta \\ \omega_3'^{(0)} \end{pmatrix} \tag{3.4}$$

$$\begin{pmatrix} \dot{\omega}_1^{(k)} \\ \dot{\omega}_2^{(k)} \\ \dot{\omega}_3^{(k)} \end{pmatrix} = A^{k0} \begin{pmatrix} \dot{\omega}_1'^{(0)} - \Omega\omega_3'^{(0)}\cos\theta \\ \dot{\omega}_2'^{(0)} + \Omega\omega_3'^{(0)}\sin\theta \\ \dot{\omega}_3'^{(0)} + \Omega\omega_1'^{(0)}\cos\theta - \Omega\omega_2'^{(0)}\sin\theta \end{pmatrix} \tag{3.5}$$

and considering of the restraint equation:

$$\dot{\omega}_1'^{(0)} = \mathrm{tg}\varphi_2 \cdot \dot{\omega}_3'^{(0)} + \omega_2'^{(0)} \cdot \omega_3'^{(0)} / \cos^2\varphi_2 \tag{3.6}$$

There are four equations in Eqs. (3.2), (3.6). Solving these four equations, we obtain $\dot{\omega}_1'^{(0)}, \dot{\omega}_2'^{(0)}, \dot{\omega}_3'^{(0)}$ and M, where M is torque.

IV. Simulation and Optimization of the Motion of Bullet Belt

Basing on above theories, and employing the computer, we can simulate the dynamics of the bullet belt numerically and graphically. The flow chart of the simulation is shown in Fig. 3.

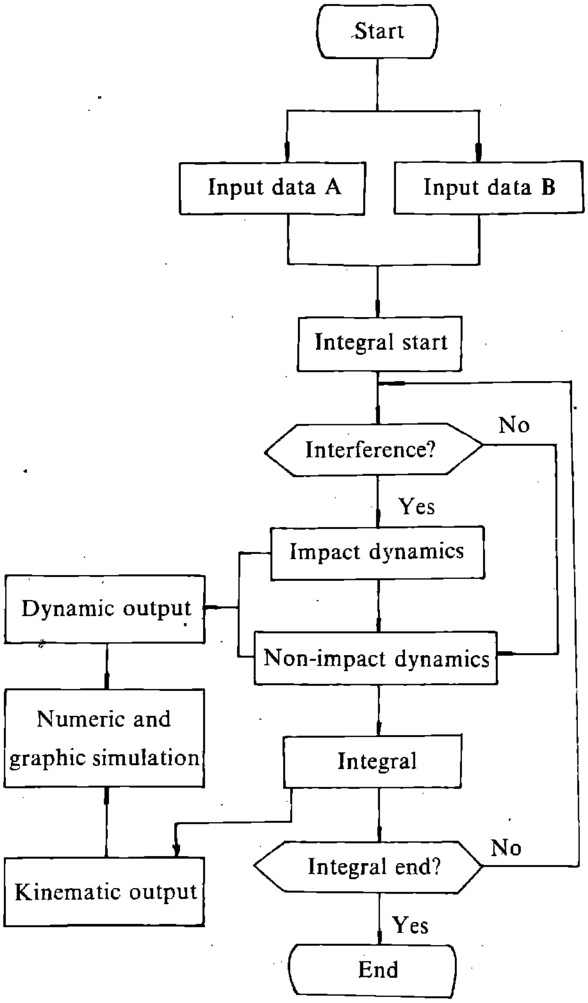

Fig. 3 The flow chart of the simulation

The laws of the motions of the bullet feeder and the gun's recoil can be measured through experiments, and they are shown separately in Fig. 4 and Fig. 5.

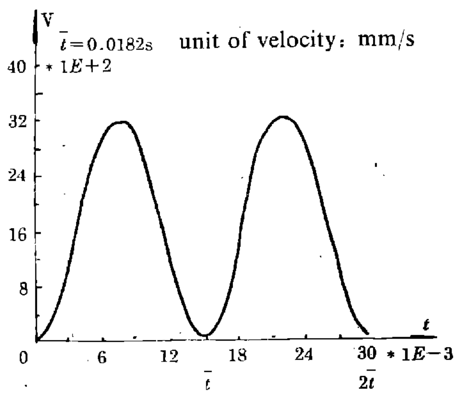

Fig. 4 The motion curve of the bullet feeder

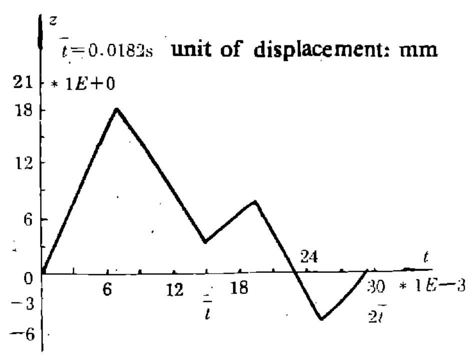

Fig. 5 The motion curve of the recoil

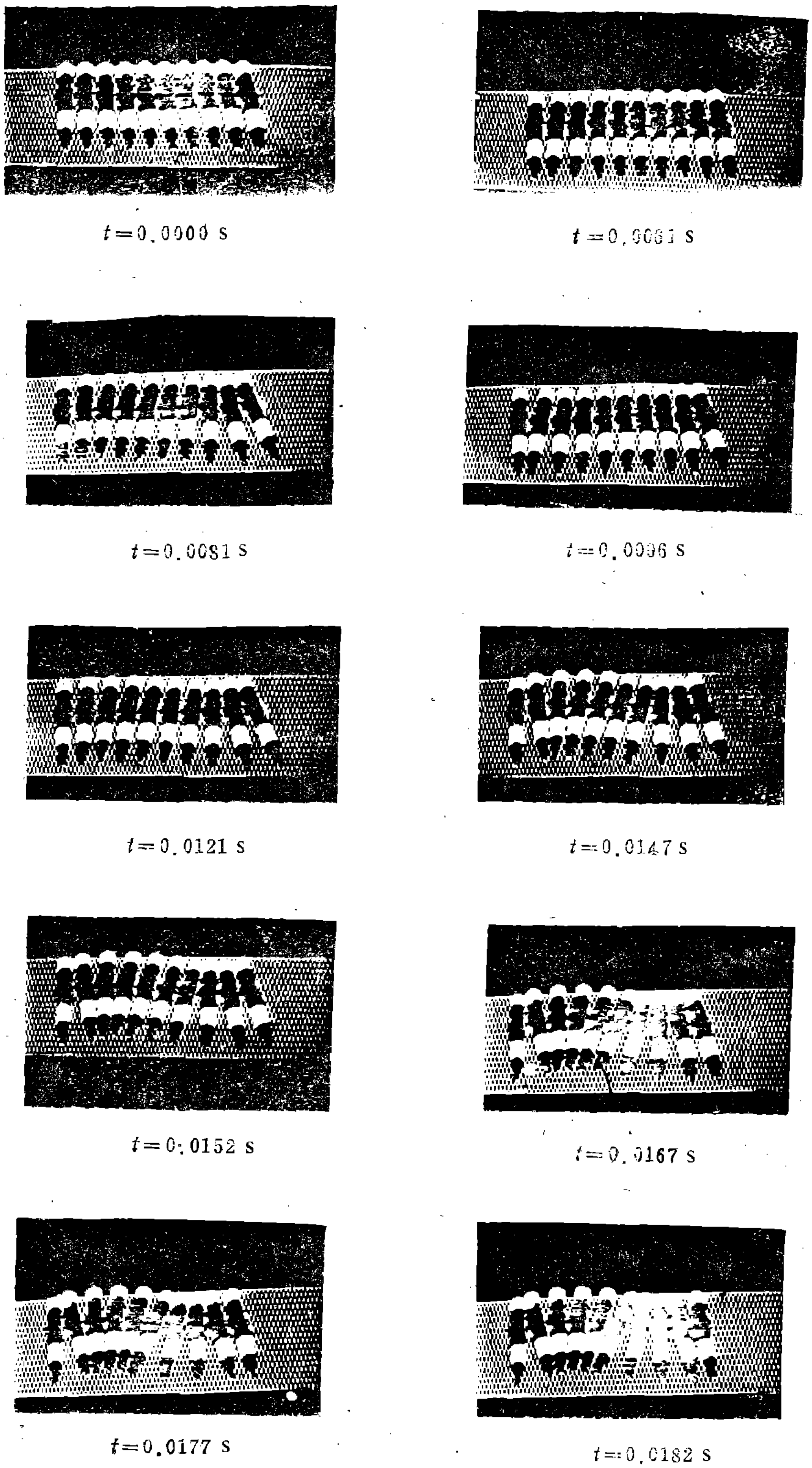

Fig. 6 Animation simulation

The simulation example is that there are total 10 bullets, from which 2 bullets are fired continually. From the computer graph animation, we can see that the motion of the bullet belt

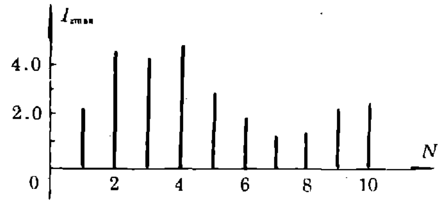

Fig. 7 The maximum of impulse of z-direction

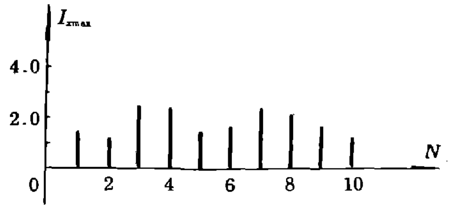

Fig. 9 The maximum of optimal impulse of z-direction

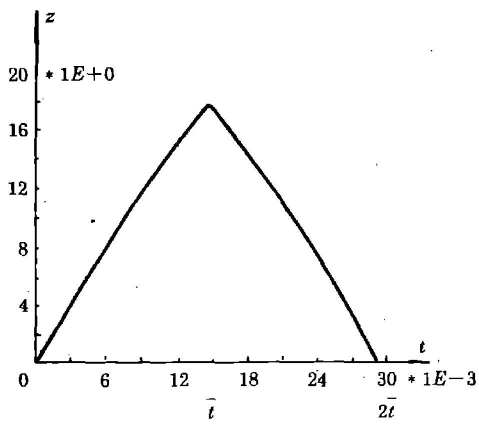

Fig. 8 The optimal curve of the recoil

is not stable, shown in Fig. 6. The bullet is loaded with a big angle φ_2 . The maximums of z-direction components of impact impulses expressed in coordinate frame $\vec{e}^{(4)}$ can be exported and shown in Fig. 7. However in general situation, the force with which the claw holds bullet is in the scope of $2.8-3.5\,\mathrm{N}$. So the bullets 2, 3, 4 and 5 have the possibility to run away.

Generally speaking, the law of the motion of the bullet feeder can not be changed easily. While the motion of the gun's recoil can be changed by way of adjusting the propeties of the buffer devices. If we adopt the motion curve of the recoil shown in Fig. 8, then we obtain the results as follows: The motion of the bullet belt is stable, and the angle φ_2 is small; The collision between bullet bodies is not very fierce, there does not occur the phenomenon of the bullet-runningaway. See Fig. 9.

V. Conclusion

The results of simulation indicate that it is inevitable for the existing system to occur the "bullet block" phenomenon. In order to reduce the number of times of "bullet block" or eliminate the "bullet block" phenomenon, some measures must be adopted to stabilize the motion of the bullet belt, so that the orientation error before loading and the amount of the running away back and forth are controlled within the limits of error. Suggesting to adopt the following methods to stabilize the motion of bullet belt:

- Adjusting the position of claw adequately;
- Optimizing the motion law of the recoil of gun.

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