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FURTHER STUDY ON LARGE AMPLITUDE VIBRATION OF CIRCULAR SANDWICH PLATES

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Abstract

In this paper, a solution of axisymmetric large amplitude vibration is presented for a circular sandwich plate with the flexure rigidity of the face layers taken into account. In solving the problem, the modified iteration method is proposed. Then our results are compared with those from paper [1].

Key words circular sandwich plate, large amplitude vibration

I. Introduction

Many people are interested in the problem of sandwich plates, but most people study only linear problem. For nonlinear problem, only a few people have done some works, because of the difficulty of solution. Liu Renhuai[2,3] has done many useful works for the problem of large deflection of sandwich plates. But, the papers on large amplitude vibration of sandwich plates are few to read. Author[1] had initial discussion to axisymmetric large amplitude vibration of circular sandwich plates of neglecting the flexure rigidity of the face layers. In this paper, we will make further study to the problem of large amplitude vibration of circular sandwich plates with the flexure rigidity of the face layers taken into account. Because the problem is boundary layer type, it is quite difficlut to solve. In solving the problem, the modified iteration method is proposed and the highest derivate term is disposed specifically, therefore the problem is simplified largely, and the results with higher precision are given.

II. Non-Dimensional Transformation of the Fundamental Equations and Boundary Conditions

The fundamental equations of axisymmetric large amplitude vibration of circular sandwich plates have been given in paper [1]

$$\left.\begin{aligned} & mrw_{,tt}+2D_f(r((rw_{,r})_{,r}/r)_{,r})_{,r}-2h_1(r\sigma_{r0}w_{,r})_{,r}-C(r(\psi+w_{,r}))_{,r}=0 \\ & \sigma_{\theta 0}-(r\sigma_{r0})_{,r}=0 \\ & D((r\psi)_{,r}/r)_{,r}-C(\psi+w_{,r})=0 \end{aligned}\right\} \tag{2.1a,b,c}$$

The stress-displacement relation is given by

$$\sigma_{r0}-(r\sigma_{\theta 0})_{,r}-Ew_{,r}^2/2=0 \tag{2.2}$$

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Then the boundary conditions of edge clamped but free from slipping are given by

when $r=a$

$$w=0,\ \psi=0,\ w_{,r}=0,\ \sigma_{r0}=0 \tag{2.3}$$

when $r=0$

$$w=w_m,\ \psi=0,\ \sigma_{r0}<\infty \tag{2.4}$$

Imitating paper [1], and introducing the non-dimensional variables

$$\left.\begin{array}{l}\rho=r/a,\ w=\bar{w}/h_0,\ S=\dfrac{3h_1a^2}{2D}\bar{S},\ \omega=\bar{\omega}(ma^4/D)^{\frac{1}{2}}\\ \lambda=\dfrac{D}{Ca^2},\ \varepsilon=\dfrac{2D_f}{Ca^2},\ \beta=\dfrac{3}{2}(1-\nu^2)\end{array}\right\} \tag{2.5}$$

we obtain

$$(\varepsilon L^3-(1+\varepsilon/\lambda)L^2-\omega^2\lambda L+\omega^2)w$$
$$-\lambda(\rho((\rho Sw_{,\rho})_{,\rho}/\rho)_{,\rho}/\rho+(\rho Sw_{,\rho})_{,\rho}/\rho=0 \tag{2.6}$$
$$((\rho^2S)_{,\rho}/\rho)_{,\rho}+\beta w_{,\rho}^2/\rho=0 \tag{2.7}$$

and the boundary conditions can be written as

when $\rho=1$

$$w=0,\ \lambda\omega^2\int_0^\rho\rho w d\rho-\varepsilon\rho((\rho w_{,\rho})_{,\rho}/\rho)_{,\rho}+\lambda\rho Sw_{,\rho}+\rho w_{,\rho}=0$$

$$w_{,\rho}=0,\ S=0 \tag{2.8a,b,c,d}$$

when $\rho=0$

$$w=w_m,\ \lambda\omega^2\int_0^\rho\rho w d\rho-\varepsilon\rho((\rho w_{,\rho})_{,\rho}/\rho)_{,\rho}+\lambda\rho Sw_{,\rho}+\rho w_{,\rho}=0$$

$$S<\infty \tag{2.9a,b,c}$$

Equations (2.6)—(2.9) are the non-dimensional fundamental equations and boundary conditions of axisymmetric large amplitude free vibration of circular sandwich plates.

III. Solution of the Modified Iteration

According to equation (2.5)

$$\frac{\varepsilon}{\lambda}=\frac{2D_f}{D}=\frac{1}{3}\left(\frac{h_1}{h_0}\right)^2$$

For the widly-used sandwich plates in engineering, we know

$$\lambda<1,\ h_1/h_0\ll 1$$

therefore

$$\varepsilon\ll 1$$

Obviously, because there is a small paremeter before the highest derivate term of equation (2.6), the problem is boundary layer type. As we know, it is more difficult to solve the problem, because the coefficient of the highest derivate term is a small parameter.

Therefore, we solve the problem by modified iteration method. In the first order approximate, we neglect the highest derivate term in equation (2.6) and the nonlinear terms in equations (2.6)−(2.9), and neglect the boundary condition (2.8c)[3], we obtain the following boundary value problem

$$((1+\varepsilon/\lambda)L^2+\omega_1^2\lambda L-\omega_1^2)w_1=0 \tag{3.1}$$

$$((\rho^2S_1)_{,\rho}/\rho)_{,\rho}+\beta w_{1,\rho}^2/\rho=0 \tag{3.2}$$

when $\rho=1$

$$w_1=0,\ \lambda\omega_1^2\int_0^\rho \rho w_1 d\rho-\varepsilon\rho((\rho w_{1,\rho})_{,\rho}/\rho)_{,\rho}+\rho w_{1,\rho}=0,\ S_1=0 \quad (3.3\text{a},\text{b},\text{c})$$

when $\rho=0$

$$w_1=w_m,\ \lambda\omega_1^2\int_0^\rho \rho w_1 d\rho-\varepsilon\rho((\rho w_{1,\rho})_{,\rho}/\rho)_{,\rho}+\rho w_{1,\rho}=0,\ S_1<\infty \quad (3.4\text{a},\text{b},\text{c})$$

The solution of equation (3.1) can be written as

$$w_1=w_m\sum_{j=0}^{\infty}A_j^{(1)}\rho^{2j} \tag{3.5}$$

where

$$A_j^{(1)}=\sum_{i=1}^{2}\mu_i\frac{a_i^j}{2^{2j}(j!)^2}$$

$$a_1=(\sqrt{\omega_1^4\lambda^2+4\omega_1^2(1+\varepsilon/\lambda)}-\omega_1^2\lambda)/2(1+\varepsilon/\lambda)$$

$$a_2=-(\sqrt{\omega_1^4\lambda^2+4\omega_1^2(1+\varepsilon/\lambda)}+\omega_1^2\lambda)/2(1+\varepsilon/\lambda)$$

where $\mu_i(i=1,2)$ is the coefficient which is to be determined by the boundary conditions. Substituting equation (3.5) into (3.3a, b) and (3.4a), we obtain

$$A\mu=0 \tag{3.6}$$

where

$$\mu=[\mu_1,\ \mu_2,\ 1]^T$$

$$A=\begin{bmatrix} a_{01} & a_{02} & a_{03} \\ a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

all of the elements of the matrix A are given by

$$a_{01}=a_{02}=-a_{03}=1,\quad a_{13}=a_{23}=0$$

$$a_{1i}=\sum_{j=0}^{\infty}\frac{a_i^j}{2^{2j}(j!)^2} \qquad (i=1,2)$$

$$a_{2i}=\sum_{j=0}^{\infty}\frac{\lambda\omega_1^2-16\varepsilon j^2(j^2-1)+2j(2j+2)}{2(j+1)\cdot 2^{2j}(j!)^2}a_i^j \qquad (i=1,2)$$

Because μ is not a zero vector, we obtain

$$\det\ A=0 \tag{3.7}$$

Solving equation (3.7), we obtain ω_1, which is the first order approximate value of fundamental frequency, and $\mu_i\,(i=1,\ 2)$ can be determined by (3.6), therefore, w_1 can also be determined. Substituting equation (3.5) into (3.2), doing integral, and appling the boundary conditions (3.3c) and (3.4c), we obtain

$$S_1=w_m^2\sum_{j=0}^{\infty}B_j^{(1)}\rho^{2j} \tag{3.8}$$

where

$$B_j^{(1)}=-\frac{\beta}{j(j+1)}\sum_{i=1}^{j}i(j-i+1)A_i^{(1)}A_{j-i+1}^{(1)}\qquad(j=1,2,\cdots)$$

$$B_0^{(1)}=-\sum_{j=1}^{\infty}B_j^{(1)}$$

In the second order iteration, we have the following modified characteristic value equations and boundary conditions

$$\begin{aligned}&((1+\varepsilon/\lambda)L^2+\omega^2\lambda L-\omega^2)w_2-\varepsilon L^2w_1\\&\quad+\lambda(\rho((\rho S_1w_{1,\rho}),_\rho/\rho),_\rho),_\rho/\rho-(\rho S,w_{1,\rho}),_\rho/\rho=0\end{aligned} \tag{3.9}$$

$$((\rho^2S_2),_\rho/\rho),_\rho+\beta w_{2,\rho}^2/\rho=0 \tag{3.10}$$

when $\rho=1$

$$w_2=0,\quad \lambda\omega^2\int_0^\rho \rho w_2d\rho-\varepsilon\rho((\rho w_{2,\rho}),_\rho/\rho),_\rho+\lambda\rho S_1w_{1,\rho}+\rho w_{2,\rho}=0,\quad S_2=0 \tag{3.11a,b,c}$$

when $\rho=0$

$$w_2=w_m,\quad \lambda\omega^2\int_0^\rho \rho w_2d\rho-\varepsilon\rho((\rho w_{2,\rho}),_\rho/\rho),_\rho+\lambda\rho S_1w_{1,\rho}+\rho w_{2,\rho}=0,\quad S_2<\infty \tag{3.12a,b,c}$$

where ω is the frequency of nonlinear vibration, that is obtained by being modified the first order approximate value of fundamental frequency, the boundary condition (2.8c) is still neglected[3].

Substitution of equations (3.5) and (3.8) into (3.9), we obtain the solution of equation (3.9), that is

$$w_2=w_m\sum_{j=0}^{\infty}A_j^{(2)}\rho^{2j}+w_m\sum_{j=0}^{\infty}B_j^{(2)}\rho^{2j}+w_m^3\sum_{j=0}^{\infty}C_j^{(2)}\rho^{2j} \tag{3.13}$$

where

$$A_j^{(2)}=\sum_{i=1}^{2}\xi_i\frac{b_i^j}{2^{2j}(j!)^2}$$

$$b_1=(\sqrt{\omega^4\lambda^2+4\omega^2(1+\varepsilon/\lambda)}-\omega^2\lambda)/2(1+\varepsilon/\lambda)$$
$$b_2=-(\sqrt{\omega^4\lambda^2+4\omega^2(1+\varepsilon/\lambda)}+\omega^2\lambda)/2(1+\varepsilon/\lambda)$$
$$B_0^{(2)}=B_1^{(2)}=0$$
$$B_{j+2}^{(2)}=\frac{D_j^{(2)}}{16(1+\varepsilon/\lambda)(j+2)^2(j+1)^2}-\frac{\omega^2\lambda}{4(1+\varepsilon/\lambda)(j+2)^2}B_{j+1}^{(2)}$$
$$+\frac{\omega^2}{16(1+\varepsilon/\lambda)(j+2)^2(j+1)^2}B_j^{(2)}$$
$$D_j^{(2)}=64\varepsilon(j+3)^2(j+2)^2(j+1)^2A_{j+3}^{(1)}$$
$$C_0^{(2)}=C_1^{(2)}=0$$
$$C_{j+2}^{(2)}=\frac{E_j^{(2)}}{16(1+\varepsilon/\lambda)(j+2)^2(j+1)^2}-\frac{\omega^2\lambda}{4(1+\varepsilon/\lambda)(j+2)^2}C_{j+1}^{(2)}$$
$$+\frac{\omega^2}{16(1+\varepsilon/\lambda)(j+2)^2(j+1)^2}C_j^{(2)}$$
$$E_j^{(2)}=-\sum_{i=1}^{j+2}16\lambda i(j+2)(j+1)^2A_i^{(1)}B_{j-i+2}^{(1)}+\sum_{i=1}^{j+1}4i(j+1)\,A_i^{(1)}\,B_{j-i+1}^{(1)}$$
$$(j=0,1,2,\cdots)$$

where ξ_i $(i=1,\ 2)$ is the coefficient which is to be determined by boundary conditions. Substituting equation (3.13) into (3.11a, b) and (3.12a), we obtain

$$\boldsymbol{C\xi}=0 \tag{3.14}$$

where

$$\boldsymbol{\xi}=[\xi_1,\ \xi_2,\ 1]^T$$
$$\boldsymbol{C}=\begin{bmatrix} C_{01} & C_{02} & f_0+w_m^2g_0 \\ C_{11} & C_{12} & f_1+w_m^2g_1 \\ C_{21} & C_{22} & f_2+w_m^2g_2 \end{bmatrix}$$

the elements C_{lm} and f_l, g_l $(l=0,\ 1,\ 2;\ m=1,\ 2)$ of matrix $\boldsymbol{C}$ can be written as infinite power series, here we will not list one by one.

Because $\boldsymbol{\xi}$ is not a zero vector, the determinant of matrix $\boldsymbol{C}$ must equal zero, that is

$$\det\boldsymbol{C}=0 \tag{3.15}$$

Equation (3.15) can be written as the following algebra equation

$$w_m^2=P(\omega)/Q(\omega) \tag{3.16}$$

where $P(\omega)$ and $Q(\omega)$ are the infinite power series, their expressions are omitted.

Let $w_m=0$ to equation (3.16), we can obtain the second order modified iteration solution of linear vibration. If let $h_1/h=0$ in preceding discussion, we obtain the results of paper [1].

Equation (3.16) is the analytic relation for amplitude-frequency response. We can obtain the value of ω corresponding to any amplitude w_m, by solving equation (3.16), and calculate

Table 1 $K=0.01$

w_m	0	1	2	3
$h_1/h=0.00$	9.4230	9.6279	10.2139	11.0537
$h_1/h=0.05$	9.4606	9.6670	10.2529	11.0928
relat. error	0.40%	0.40%	0.38%	0.35%
$h_1/h=0.10$	9.5017	9.7158	10.2920	11.1318
relat. error	0.83%	0.90%	0.76%	0.70%
$h_1/h=0.15$	9.5450	9.7549	10.3408	11.1807
relat. error	1.28%	1.30%	1.23%	1.14%
$h_1/h=0.20$	9.5893	9.7939	10.3799	11.2197
relat. error	1.73%	1.70%	1.60%	1.48%

Table 2 $K=0.03$

w_m	0	1	2	3
$h_1/h=0.00$	8.2611	8.4658	9.0029	9.7646
$h_1/h=0.10$	8.3343	8.5342	9.0811	9.8428
relat. error	0.88%	0.80%	0.86%	0.79%
$h_1/h=0.10$	8.4087	8.6123	9.1592	9.9209
relat. error	1.76%	1.70%	1.71%	1.57%
$h_1/h=0.15$	8.4833	8.6807	9.2373	9.9990
relat. error	2.62%	2.47%	2.54%	2.34%
$h_1/h=0.20$	8.5574	8.7588	9.3057	10.0772
relat. error	3.46%	3.34%	3.25%	3.10%

Table 3 $K=0.05$

w_m	0	1	2	3
$h_1/h=0.00$	7.4396	7.6357	8.1631	8.8662
$h_1/h=0.05$	7.5287	7.7236	8.2607	8.9639
relat. error	1.18%	1.14%	1.18%	1.09%
$h_1/h=0.10$	7.6185	7.8213	8.3486	9.0615
relat. error	2.35%	2.37%	2.22%	2.16%
$h_1/h=0.15$	7.7080	7.9092	8.4365	9.1592
relat. error	3.48%	3.46%	3.24%	3.20%
$h_1/h=0.15$	7.7966	7.9971	8.5244	9.2471
relat. error	4.58%	4.52%	4.24%	4.12%

Table 4 $K=0.1$

w_m	0	1	2	3
$h_1/h=0.00$	6.1256	6.3271	6.8447	7.4697
$h_1/h=0.05$	6.2256	6.4248	6.9424	7.5771
relat. error	1.61%	1.52%	1.41%	1.42%
$h_1/h=0.10$	6.3270	6.5322	7.0400	7.6846
relat. error	3.18%	3.14%	2.77%	2.80%
$h_1/h=0.15$	6.4284	6.6299	7.1475	7.7920
relat. error	4.71%	4.57%	4.24%	4.14%
$h_1/h=0.20$	6.5290	9.7275	7.1475	7.8994
relat. error	6.18%	5.95%	4.24%	5.44%

the values of $A_j^{(2)}$, $B_j^{(2)}$, $C_j^{(2)}$ by given formula. Thus elements C_{lm} and f_l, g_l of matrix $\mathbf{C}$ are determined, and the values of ξ_1, ξ_2 can be determined by equation (3.14). So far, the solution of the second order modified iteration of boundary value problems (2.6)−(2.9) is determined completely.

We complete the numerical calculation for discussed circular sandwich plate with edge clamped but free from slipping, the results are given in Tables 1−4, and the results are compared with those from paper [1], where $K=D/G_2h_0a^2$ is the shearing parameter which is given in paper [1], and the Poisson ratio $\nu=0.3$.

IV. Conclusion

We have discussed the problem of axisymmetric large amplitude free vibration of circular sandwich plates with the flexure rigidity of the face layers taken into account, by the modified iteration method. According to the preceding analysis and results, we know:

1. In this paper, the flexure rigidity of the face layers taken into account, the only approximation is to be neglected a boundary condition of face layers, and, so is paper [1], therefore, the results are more accurate than those of paper [1].

2. Because the highest derivate term with a small parameter is disposed specifically, the solution of the problem is easier.

3. The relative errors which are caused by neglecting the flexure rigidity of the face layers are increased gradually with the increase of h_1/h value. When $0.1<h_1/h<0.2$, the largest relative error is less than 6.2%, therefore, when h_1/h is smaller, we can apply the approximate theory in paper [1].

4. When K is smaller, the results of paper [1] are near those from this paper, therefore, the effects of the flexure rigidity of the face layers not only vary with h_1/h, but also vary with the shearing parameters of sandwich plates.

5. The relative errors which are caused by neglecting the flexure rigidity of the face layers vary with w_m, but the variety is very small.

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