

TWO-PHASE FLOW FOR A HORIZONTAL WELL PENETRATING A NATURALLY FRACTURED RESERVOIR WITH EDGE WATER INJECTION

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Abstract

This paper examines the two-phase flow for a horizontal well penetrating a naturally fractured reservoir with edge water injection by means of a fixed streamline model. The mathematical model of the vertical two-dimensional flow of oil-water for a horizontal well in a medium with double-porosity is established, and whose accurate solutions are obtained by using the characteristic method. The saturation distribution in the fractured system and the matrix system as well as the formula of the time of water free production are presented. All these results provide a theoretical basis and a computing method for oil displacement by edge water from naturally fractured reservoirs.

Key words horizontal well, two-phase flow, medium with double-porosity, edge water injection, characteristic method

I. Introduction

The horizontal well is of broad prospect to exploit oil or gas or water reservoirs, whose fluid mechanics basis is the vertical two-dimensional flow of oil-water. Therefore it is of important meaning theoretically and actually to research the two-dimensional flow of the two-phase fluid for a horizontal well. The basic theory on one-dimensional flow of oil-water for a vertical well was established by Buckley and Leverett^[1], and was generalized into a medium with double-porosity by Chen Zhongxiang and Liu Ciqun^[2]. With respect to the two-dimensional flow of the two-phase fluid for a horizontal well in a porous medium, Liu Ciqun^[3] had a study by means of a fixed streamline model and obtained some conclusions. This paper discusses the two-dimensional flow of the two-phase fluid for a horizontal well penetrating a naturally fractured reservoir with edge water injection. This work can be considered as an extension and a perfection of these results in Ref. [2] and [3].

II. Equations for Two-Dimensional Flow of Oil-Water

Suppose that a horizontal well with the horizontal length L is located on the top of a naturally fractured reservoir with the thickness h (see Fig. 1). It is considered to inject water at

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the beginning $t=0$ and the position $x=x_e$. Let $S_{wf}(x, z)$ and $S_{wm}(x, z)$ represent the initial water saturation distributions in the fractured system and the matrix system respectively. The above problem can be reduced to solve the following equations for the vertical two-dimensional flow of the two-phase fluid, provided that the actions of capillary and gravity are not counted.

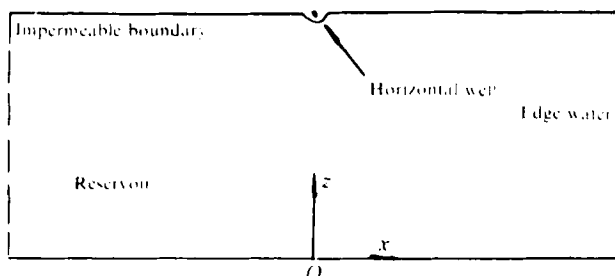


Fig. 1 Vertical cross-section of the horizontal well in the naturally fractured reservoir

For the fractured system, we have

$$\phi_f \frac{\partial S_{wf}}{\partial t} + V_z f'_{wf}(S_{wf}) \frac{\partial S_{wf}}{\partial x} + V_z' f'_{wf}(S_{wf}) \frac{\partial S_{wf}}{\partial z} = q_w \quad (2.1)$$

$$q_w = R\lambda \left[\lambda \int_0^t S_{wf}(x, z, \tau) e^{-\lambda(t-\tau)} d\tau - S_{wf} \right] \quad (2.2)$$

$$S_{wf} = S_{wf}(x, z, t) \Big|_{t=0}^{\Delta} = \begin{cases} S_{wf0}(x, z), & t=0 \\ 1, & x=x_e \end{cases} \quad (2.3)$$

For the matrix system, we have

$$\phi_m \frac{\partial S_{wm}}{\partial t} = -q_w \quad (2.4)$$

$$S_{wm} = S_{wm0}(x, z), \quad t=0 \quad (2.5)$$

where

$$f'_{wf}(S_{wf}) = \mu_f [(1-\mu)S_{wf} + \mu]^{-1} \quad (2.6)$$

S , ϕ and q represent the saturation, the porosity and the imbibition intensity respectively. μ is the ratio of viscosity of water over that of oil. R is the volume of the oil that ooze from a unit volume of matrix under the action of imbibition. λ is a constant that expresses the declining rate of imbibition. The subscripts w , f and m indicate the physical quantity of water, that of the fractured system and that of the matrix system respectively. V_x and V_z are respectively the components of the flow velocity along the direction of x and z . Assume that the streamline is fixed, and that the flow rate of the horizontal well is Q , the flow velocity are given^[1] as following

$$V_x = -\frac{Q}{2hL} \cdot \frac{\text{sh}(\pi x/h)}{\text{ch}(\pi x/h) + \cos(\pi z/h)} \quad (2.7)$$

$$V_z = \frac{Q}{2hL} \cdot \frac{\sin(\pi z/h)}{\text{ch}(\pi x/h) + \cos(\pi z/h)} \quad (2.8)$$

III. Accurate Solutions of the Equations

(1) Solutions for the fractured system

According to the theory of characteristic, we know that the characteristic equations of the problem (2.3) to the quasilinear hyperbolic Eq. (2.1) and the imbibition Eq. (2.2) are as follows

$$\frac{dx}{dt} = \frac{V_s}{\phi_f} \cdot \frac{\mu}{[(1-\mu)S_{wf} + \mu]^2} \quad (3.1)$$

$$\frac{dz}{dt} = \frac{V_s}{\phi_f} \cdot \frac{\mu}{[(1-\mu)S_{wf} + \mu]^2} \quad (3.2)$$

For any x_0, z_0, t_0 , $x = x(x_0, z_0, t_0; t)$ and $z = z(x_0, z_0, t_0; t)$ represent the characteristic curve passing through the point $(x_0, z_0, 0)$ or the point (x_0, z_0, t_0) , we define the function $S_{wf}(t)$ by

$$S_{wf}(t) \triangleq S_{wf}(x(x_0, z_0, t_0; t), z(x_0, z_0, t_0; t), t) \quad (3.3)$$

Thus it satisfies

$$\frac{dS_{wf}(t)}{dt} = \frac{R\lambda}{\phi_f} \left[\lambda \int_0^t S_{wf}(\tau) e^{-\lambda(t-\tau)} d\tau - S_{wf}(t) \right] \quad (3.4)$$

Differentiating (3.4) on both sides in t , the Eq. (3.4) becomes a second order homogeneous linear differential equation with constant coefficient. Solving this equation, along the characteristic curve passing through the point $(x_0, z_0, 0)$ we have

$$S_{wf}(t) = S_{wf}(x_0, z_0, 0) \frac{\phi_f + R \exp \left[-\frac{(\phi_f + R)\lambda}{\phi_f} \cdot t \right]}{\phi_f + R} \quad (3.5)$$

Let the characteristic curve passing through the point (x_0, z_0, t_0) intersects the plane $t=0$. Then along the characteristic curve passing through the point $(x_0, z_0, 0)$ or the point (x_0, z_0, t_0) we have

$$S_{wf}(t) = S_{wf}(x_0, z_0, t_0) \frac{\phi_f + R \exp \left[-\frac{(\phi_f + R)\lambda}{\phi_f} \cdot t \right]}{\phi_f + R \exp \left[-\frac{(\phi_f + R)\lambda}{\phi_f} \cdot t_0 \right]} \quad (3.6)$$

In order to get the expressions of characteristic curve, we first consider the problem

$$V_s dz - V_s dx = 0 \quad (3.7)$$

Substituting for (2.7) and (2.8) in the Eq. (3.7), we get the following formula of streamline distributions

$$\text{ctg}(\pi z/2h) = c \cdot \text{th}(\pi x/2h) \quad (3.8)$$

where

$$c = \text{ctg}(\pi z_0/2h) \cdot \text{th}(\pi x_0/2h) \quad (3.9)$$

After giving the formula of streamline distributions (3.8), we only need to get the expression of the characteristic curve $x = x(x_0, z_0, t_0; t)$. Substituting for (2.7), (3.6) and (3.8) in the Eq. (3.1), we can deduce that the expression of characteristic curve $x = x(x_0, z_0, t_0; t)$ passing through the point $(x_0, z_0, 0)$ or the point (x_0, z_0, t_0) is as following

$$\begin{aligned}
& \ln \left[(1+c^2) \operatorname{ch} \frac{\pi x_0}{h} + (1-c^2) \right] - \ln \left[(1+c^2) \operatorname{ch} \frac{\pi x}{h} + (1-c^2) \right] \\
&= \frac{\pi Q \mu}{2h^2 L \lambda (\phi_f + R) [(1-\mu) S_{wf}(\infty) + \mu]^2} \cdot \left[\frac{(\phi_f + R) \lambda}{\phi_f} \cdot (t-t_0) \right. \\
&\quad + \frac{(1-\mu) S_{wf}(\infty) + \mu}{(1-\mu) S_{wfi}(x_0, z_0, t_0) + \mu} - \frac{(1-\mu) S_{wf}(\infty) + \mu}{(1-\mu) S_{wf}(t) + \mu} \\
&\quad \left. + \ln \frac{(1-\mu) S_{wf}(t) + \mu}{(1-\mu) S_{wfi}(x_0, z_0, t_0) + \mu} \right] \quad (3.10)
\end{aligned}$$

Let $x=x_0$, $x=0$, $z_0=h$, $t_0=0$ and $S_w(x, z)=0$ in the above expression, we get the following formula of the time of water free production

$$T = \frac{4h^2 L \phi_f \mu}{\pi Q} \cdot \ln \operatorname{ch} \frac{\pi x_0}{2h} \quad (3.11)$$

The above formula means that the time of water free production for a horizontal well is in direct proportion to the square of the reservoir thickness h , is also in direct proportion to the horizontal length L , is in inverse proportion to the flow rate Q .

(2) Solutions for the matrix system

For one thing, let $x=x_0$ in the Eq. (2.4), we can get the expression of $S_w(x_0, z, t)$. Hence the initial problem (2.5) of the Eq. (2.4) can be written as the following initial and boundary problem of that

$$S_{wm} = S_{wmi}(x, z, t) \triangleq \begin{cases} S_{wmc}(x, z), & t=0 \\ S_{wmc}(x_0, z) + \frac{R}{\phi_m} \cdot (1-e^{-\lambda t}), & x=x_0 \end{cases} \quad (3.12)$$

For another, we integrate (2.4) on both sides in t . Using integration by parts, we can deduce the following equivalent equation with only instantaneous quantities

$$q_w = \lambda \phi_m (S_{wm} - S_{wmc}) - R \lambda S_{wf} \quad (3.13)$$

Finally, substituting for (3.5) in the expression (3.13), we let the characteristic curve passing through the point (x_0, z, t) intersects the plane $t=0$. Therefore along the characteristic curve passing through the point $(x, z, 0)$ or the point (x_0, z, t) we have

$$\begin{aligned}
S_{wm}(t) &= S_{wmi}(x_0, z_0, t_0) + S_{wfi}(x_0, z_0, t_0) \cdot \frac{\phi_f R}{\phi_m} \\
&\quad \cdot \frac{\exp \left[-\frac{(\phi_f + R) \lambda}{\phi_f} \cdot t_0 \right] - \exp \left[-\frac{(\phi_f + R) \lambda}{\phi_f} \cdot t \right]}{\phi_f + R \exp \left[-\frac{(\phi_f + R) \lambda}{\phi_f} \cdot t_0 \right]} \quad (3.14)
\end{aligned}$$

IV. Calculation of an Example

Using these results mentioned above, we give an example. The physical model is shown in Fig. 1, whose parameters are as follows: $h=40\text{m}$, $L=500\text{m}$, $Q=100\text{m}^3/\text{d}$, $x_0=100\text{m}$, $\phi_f=0.01$, $\phi_m=0.1$, $S_w(x, z)=0$, $S_{wc}(x, z)=0.2$, $\mu=0.1$, $R=0.06$, $\tau^*=1000\text{d}$, $\lambda=\ln 2 \cdot \tau^* = 6.9315 \times 10^{-4}\text{d}^{-1}$. Fig. 2-5 show some results.

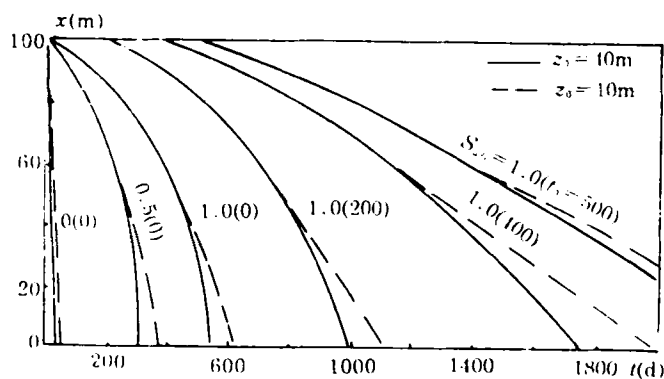


Fig. 2 Characteristic curves along different streamlines

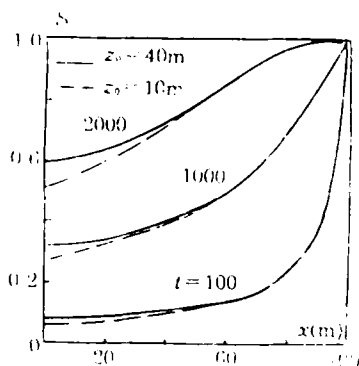


Fig. 3 Saturation variations in the fractured system

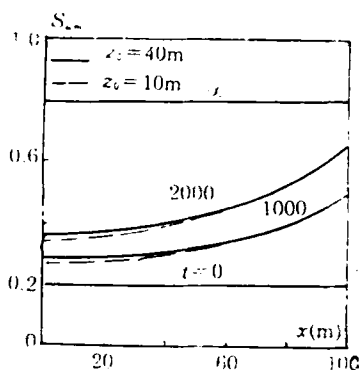


Fig. 4 Saturation variations in the matrix system

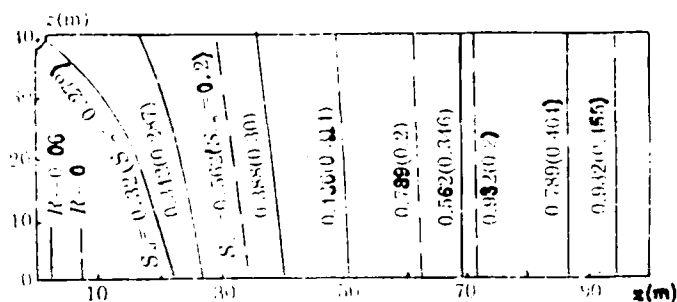


Fig. 5 Saturation distributions of $t = 1000d$

V. Conclusions

(1) The mathematical model of the vertical two-dimensional flow of oil-water for a horizontal well in a medium with double-porosity is established, and the accurate solutions are obtained by using the method of characteristic. The saturation distributions in the fractured system and the matrix system as well as the formula of the time of water free production are presented. All these results provide a theoretical basis and a computing method for oil displacement by edge water from naturally fractured reservoirs.

(2) The essential nature of the two-dimensional flow of oil-water for a horizontal well along every streamline is similar to that of the one-dimensional flow of the two-phase fluid for a vertical well. But the value of saturation vary differently along different streamline. The issaturation curves in the fractured system and the matrix system will move toward the horizontal well with the time passing.

(3) The time of water free production for a horizontal well is in direct proportion to the square of the reservoir thickness, is also in direct proportion to the horizontal length, is in inverse proportion to the flow rate.

(4) The one-dimensional flow of oil-water for a vertical well in a medium with double-porosity and the two-dimensional flow of the two-phase fluid for a horizontal well in a porous medium are two special cases of this work. By using our method of analysis and calculation, it is not difficult to obtain the accurate solutions on the former and have an overall study on the latter. Therefore this work can be considered as an extension and a perfection of these results in Refs. [2] and [3].

References

- [1] S. E. Buckley and M. C. Leverett, Mechanism of fluid displacement in sands, *Trans. AIME*, **146** (1942), 107–116.
- [2] Chen Zhongxiang and Liu Ciqun, Theory of fluid displacement in a medium with double-porosity, *Acta Mechanica Sinica*, **2** (1980), 109–119. (in Chinese)
- [3] Liu Ciqun, Two-phase flow for a horizontal well, *Mechanics and Practice*, **2** (1993), 23–24. (in Chinese)