

ANALYSIS OF STABILITY ON ELASTIC PLATES WITH INITIAL IMPERFECTIONS

Xu Kaiyu (徐凯宇)¹

(Received Oct. 5, 1994; Communicated Pai Lizhou)

Abstract

On the basis of Marguerre equations, the influence on a bifurcation diagram of an elastic plate affected by initial deflection imperfection and transverse loading is studied with the help of the singularity theory. This paper applies universal unfolding principles, it is put forward that the unstable analysis of this problem can transform into the study of a triple algebraic equation in the neighborhood of a simple eigenvalue. Thus the bifurcated states are decided, and the bifurcation diagrams are drawn up following distinct parameters. Then the quantitative series of interfering with eigenvalues are discussed.

Key words imperfection, universal unfolding, eigenvalue, bifurcation

I. Introduction

In varying degrees there are imperfections of all kinds in actual structures for a variety of reasons. An ideal and perfect structure only has theoretical significance. If there are some initial imperfections in a structure, the equilibrium is seldom a bifurcation problem, the structure is normally regarded as instability at extreme point^[1], and critical load is sensitively dependent on the initial imperfections when extreme point exists. Another situation is that the unstable problem of a structure is changed into a bending problem under the influence of imperfection^{[2], [3]}. For a long time in the past, the study of stability of structure with imperfections is a continuously brisk field. The nonlinear equations with regard to an elastic plate with initial deflection were formulated by Marguerre^[4] first. Now, they are widely applied to analysis of overload capacity in post-buckled plate and nonlinear behavior of shallow shell. A large number of numerical calculus is mainly concentrated on finite element method and variational method^{[5], [6]}. In qualitative analysis respect, Poston and Stewart^[7]. Have studied the buckling state of a flat pressed plate with transverse load by using catastrophe theory. Gaspar^[8] has discussed the global bifurcation solutions of a plate with initial deflection.

The arbitrary shape elastic plate with initial deflection and transverse load is considered in this paper. Using the singularity theory^[9], the buckling state of the plate clamped and pressed on its boundary is researched. Lastly, some new analysis results are obtained.

¹ Shanghai University; Shanghai Institute of Applied Mathematics and Mechanics, Shanghai 200072, P. R. China

II. Formulations of Governing Equations and Basic Properties

Suppose a thin elastic plate occupies $\Omega \subset R^2$, at initial position, the plate is not completely located in x - y plane if there is no external applied forces. This small imperfection is a deflection $\alpha w_0(x, y)$, $\alpha \in R$, w_0 is a known function, the transverse loading is indicated by $\beta q_0(x, y)$, $\beta \in R$. Let the plate's boundary be clamped and borne a horizontal press force $\lambda \psi(x, y)$, $(x, y) \in \partial\Omega$, $\lambda \in R$. In that case, the stress function $f(x, y)$ and the additional deflection $w(x, y)$ satisfy non-dimensional Marguerre equations and corresponding boundary conditions as follows:

$$\left. \begin{aligned} \Delta^2 f &= -\frac{1}{2}[w, w] - \alpha[w_0, w] \\ \Delta^2 w &= [w + \alpha w_0, f + \lambda F_0] + \beta q_0 \end{aligned} \right\} \quad (\Omega) \quad (2.1)$$

$$f = f_n = f_t = 0, \quad w = w_n = w_t = 0 \quad (\partial\Omega) \quad (2.2)$$

in which $F_0(x, y)$ is the solution of differential equation

$$\Delta^2 F_0 = 0 \quad (\Omega) \quad (2.3)$$

$$F_{0,nn} = \psi(n), \quad F_{0,nt} = \psi(t) \quad (\partial\Omega) \quad (2.4)$$

where $\psi(n)$, $\psi(t)$ are the projection components of ψ along normal and tangent directions of the boundary curve, Δ^2 is a biharmonic operator, the square brackets $[\cdot, \cdot]$ are defined as

$$[u, v] = u_{,xx}v_{,yy} + u_{,yy}v_{,xx} - 2u_{,xy}v_{,xy} \quad (2.5)$$

In order to formulate operator equations, let's introduce a Hilbert space

$$H = \left\{ u \in W_0^{1,2}(\Omega) \mid \langle u, v \rangle = \int_{\Omega} \Delta u \cdot \Delta v \quad \forall u, v \in W_0^{1,2}(\Omega) \right\}$$

$\forall \varphi, \eta \in H$, multiplied by Eq. (2.1), and then the equations are integrated using the boundary conditions (2.2), we obtain

$$\int_{\Omega} \Delta^2 f \cdot \varphi = \langle f, \varphi \rangle, \quad \int_{\Omega} \Delta^2 w \cdot \eta = \langle w, \eta \rangle, \quad \int_{\Omega} [f, w] \cdot \eta = b(f, w, \eta)$$

in which

$$b(u, v, \varphi) = \int_{\Omega} [(u_{,xx}v_{,yy} - u_{,yy}v_{,xx})\varphi_x + (u_{,xx}v_{,xx} - u_{,xx}v_{,yy})\varphi_y] \quad (2.6)$$

Note $c_1(u, \varphi) = b(F_0, u, \varphi)$, $c_2(u, \varphi) = b(w_0, u, \varphi)$, according to Riesz representation theorem, there are bilinear operator B , $\langle B(u, v), \varphi \rangle = b(u, v, \varphi)$, $\forall \varphi \in H$, and linear operators A, G, g , causing

$$\langle Au, \varphi \rangle = c_1(u, \varphi), \quad \langle Gu, \varphi \rangle = c_2(u, \varphi), \quad \langle g, \varphi \rangle = \int_{\Omega} q_0 \varphi$$

So the solutions of governing Eqs. (2.1), (2.2) satisfy

$$\left. \begin{aligned} \langle f, \varphi \rangle &= -\frac{1}{2}b(w, w, \varphi) - \alpha c_2(w, \varphi) \\ \langle w, \eta \rangle &= b(w, f, \eta) + \lambda c_1(w, \eta) + \alpha c_2(f, \eta) + \alpha \lambda c_1(w_0, \eta) + \beta \langle g, \eta \rangle \end{aligned} \right\} \quad (2.7)$$

Substituting b, c_1, c_2 into formulas (2.7) and noticing the $\varphi, \eta \in H$ are arbitrary, we have

$$f = -\frac{1}{2}B(w, w) - \alpha Gw \quad (2.8)$$

$$w - \lambda Aw + \alpha^2 G^2 w + \alpha Q(w) + C(w) = \alpha \lambda p + \beta g \quad (2.9)$$

where

$$Q(w) = B(w, Gw) + \frac{1}{2}GB(w, w) \quad (2.10)$$

$$C(w) = \frac{1}{2}B(w, B(w, w)) \quad (2.11)$$

$$p = Aw_0 \quad (2.12)$$

The final outcome is that the boundary value problem can be included in solving two non-coupling operator Eqs. (2.8) and (2.9).

To further discuss, we will study some properties of the operator equations. Berger^[10] has proved

1. $B: H \times H \rightarrow H$ is a bounded bilinear operator, moreover,

$$\langle B(u, v), w \rangle = \langle B(w, v), u \rangle \quad (2.13)$$

2. For fixed $u_0 \in H$, $B(u_0, v): H \rightarrow H_0$ is a compact operator.

3. A is a self-adjoint bounded linear operator.

4. $C(\sigma w) = \sigma^3 C(w) \quad \forall \sigma \in R \quad (2.14)$

$$\langle C(w), w \rangle = \|B(w, w)\|^2 \quad (2.15)$$

$$\langle C(w), w \rangle \geq 0, \quad \langle C(w), w \rangle = 0 \iff w = 0 \quad (2.16)$$

Since $\langle Gu, \varphi \rangle = \langle B(w_0, u), \varphi \rangle$, we immediately obtain

5. G and G are self-adjoint bounded linear operators.

With regard to the nonlinear terms in operator Eq. (2.9), the properties are as follows.

6. $Q(w)$ is a quadric homogeneous operator, namely,

$$Q(\sigma w) = \sigma^2 Q(w) \quad (2.17)$$

In fact, by (2.10), we obtain

$$\begin{aligned} Q(\sigma w) &= B(\sigma w, G\sigma w) + \frac{1}{2}GB(\sigma w, \sigma w) \\ &= \sigma^2 B(w, Gw) + \frac{1}{2}\sigma^2 GB(w, w) = \sigma^2 Q(w) \end{aligned}$$

III. Influence of Imperfection Parameter and Transverse Loading on Bifurcation Diagram

We will first study the problem of solution of Eq. (2.9). If w is solved from Eq. (2.9), f can be computed by substituting w into (2.8). Now we begin to research into a perfect plate's linear eigenvalue problem. By linearization of Eq. (2.9), we obtain^[11]

$$(I - \lambda A)w = 0, \quad w \in H \quad (3.1)$$

Let $1/\lambda_0$ be an eigenvalue of A , then $\lambda_0 \neq 0$ ^[12], its mechanics meaning is critical loading. For convenience' sake, we make a change of variable $\mu = \lambda - \lambda_0$, then the Eq. (2.9) transforms into

$$\begin{aligned} w - (\lambda_0 + \mu)Aw + \alpha^2 G^2 w + \alpha Q(w) \\ + C(w) - \alpha(\lambda_0 + \mu)p - \beta g = 0 \end{aligned} \quad (3.2)$$

Furthermore, let $1/\lambda_0$ be a simple eigenvalue of A , that is

$$\dim \ker(I - \lambda_0 A) = 1 = \dim \operatorname{coker}(I - \lambda_0 A) \quad (3.3)$$

If $P: H \rightarrow H$ is a projecting mapping, $\operatorname{Range}(P) = \ker(I - \lambda_0 A)$, by Lyapunov-Schmidt procedure, $\forall w \in H$, $w = Pw + (I - P)w$, for $e_1 \in \ker(I - \lambda_0 A)$, $\|e_1\| = 1$, then the Eq. (3.2) can transform into

$$\begin{aligned} k(u, \mu, \alpha, \beta)e_1 \equiv & P(I - (\lambda_0 + \mu)A)(ue_1 + v^*(u, \mu, \alpha, \beta)) + P(\alpha^2 G^2(ue_1 \\ & + v^*(u, \mu, \alpha, \beta))) + P(\alpha Q(ue_1 + v^*(u, \mu, \alpha, \beta))) \\ & + P(C(ue_1 + v^*(u, \mu, \alpha, \beta))) - P(\alpha(\lambda_0 + \mu)p) - P(\beta g) = 0 \end{aligned} \quad (3.4)$$

in which

$$v = v^*(u, \mu, \alpha, \beta) = O(|u|^3 + |\alpha||u|^2) \quad (3.5)$$

Therefore, the solutions of Eq. (3.2) are equivalent to solving an algebraic equation

$$k: R \times R \times R \times R \rightarrow R, \quad k(u, \mu, \alpha, \beta) = 0 \quad (3.6)$$

Notice $\forall w \in H$, $Pw = \langle w, e_1 \rangle e_1$, and $\langle v^*, e_1 \rangle = 0$, $A(ue_1) = uAe_1 = ue_1/\lambda_0$, recall A is self-adjoint, we have

$$\langle Av^*, e_1 \rangle = \langle v^*, Ae_1 \rangle = \langle v^*, e_1/\lambda_0 \rangle = 0$$

that is $Av^* \in \{\operatorname{span}(e_1)\}^\perp$, $P(Av^*) = 0$, the right side of (3.4) can be written

$$\begin{aligned} \langle C(ue_1 + v^*), e_1 \rangle + \alpha \langle Q(ue_1 + v^*), e_1 \rangle + (\alpha^2 \langle G^2 e_1, e_1 \rangle - \mu/\lambda_0)u \\ - \alpha(\lambda_0 + \mu)\langle p, e_1 \rangle - \beta \langle g, e_1 \rangle + \alpha^2 \langle G^2 v^*, e_1 \rangle = 0 \end{aligned} \quad (3.7)$$

Considering property 5 in Section II, we can compute

$$\langle G^2 e_1, e_1 \rangle = \langle Ge_1, Ge_1 \rangle = \|Ge_1\|^2 \geq 0 \quad (3.8)$$

On the basis of properties of C , Q and G as well as (3.5), we write Eq. (3.6) finally

$$\begin{aligned} k(u, \mu, \alpha, \beta) = C_0 u^3 + \alpha C_1 u^2 + \left(\alpha^2 C_2 - \frac{\mu}{\lambda_0} \right) u - \alpha(\lambda_0 + \mu)C_3 \\ - \beta C_4 + \operatorname{hot.} = 0 \end{aligned} \quad (3.9)$$

where

$$\begin{aligned} C_0 &= \langle Ce_1, e_1 \rangle, \quad C_1 = \langle Qe_1, e_1 \rangle, \quad C_2 = \langle G^2 e_1, e_1 \rangle \geq 0 \\ C_3 &= \langle p, e_1 \rangle = \langle Aw_0, e_1 \rangle = \frac{1}{\lambda_0} \langle w_0, e_1 \rangle, \quad C_4 = \langle g, e_1 \rangle \\ \operatorname{hot.} &= O(|u|^4 + (|\alpha|^2 + |\mu|)u^2 + (|\mu|^2 + |\alpha|^3 + |\beta|)|u| \\ &\quad + (|\alpha| + |\mu| + |\beta|)^2) \end{aligned}$$

when $\alpha = \beta = 0$, the Eq. (3.9) returns to the bifurcation equation of stability analysis for a flat plate

$$k^0(u, \mu) = k(u, \mu, 0, 0) = C_0 u^3 - \frac{\mu}{\lambda_0} u + \text{hot.} = 0 \quad (3.10)$$

$k^0(u, \mu)$ satisfying

$$\left. \begin{aligned} k^0(0, 0) = 0, \quad k_u^0(0, 0) = 0, \quad k_{uu}^0(0, 0) = 0, \quad k_{\mu}^0(0, 0) = 0 \\ k_{\mu\mu}^0(0, 0) = 6C_0 > 0, \quad k_{\mu\mu}^0(0, 0) = -\frac{1}{\lambda_0} \neq 0 \end{aligned} \right\} \quad (3.11)$$

The abovementioned formulas are computed by using property of C in (2.16). So the conditions for Theorem 3.1.2 in literature [13] are satisfied, we have the following result.

Theorem 3.1 $(u, \lambda) = (0, \lambda_0)$ is a bifurcation point for Eq. (3.10), and the bifurcation diagrams are completely decided by

$$C_0 u^3 - \frac{\mu}{\lambda_0} u = 0 \quad (3.12)$$

Here, we have proved again that buckling state of an elastic flat plate must arise at the simple eigenvalue.

In order to study initial imperfections and transverse loading affect on the solution of pitchfork, we need to consider a universal unfolding of the left side in (3.12).

Theorem 3.2 $F(u, \mu, \alpha, \beta) = C_0 u^3 + \alpha C_1 u^2 - \frac{\mu}{\lambda_0} u - \alpha \lambda_0 C_3 - \beta C_4$ is a universal unfolding of $h(u, \mu) = C_0 u^3 - \mu u / \lambda_0$.

Proof h satisfies condition (3.11) at $(u, \mu) = (0, 0)$. F is a 2-parameter unfolding, and

$$\det \begin{bmatrix} h_u & h_{uu} & h_{u\mu} & h_{uuu} \\ h_{\mu} & h_{\mu u} & h_{\mu\mu} & h_{\mu uu} \\ F_{t_1} & F_{t_1 u} & F_{t_1 \mu} & F_{t_1 uu} \\ F_{t_2} & F_{t_2 u} & F_{t_2 \mu} & F_{t_2 uu} \end{bmatrix}_{(0,0)} = \det \begin{bmatrix} 0 & 0 & -\frac{1}{\lambda_0} & 6C_0 \\ 0 & -\frac{1}{\lambda_0} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} = -\left(\frac{1}{\lambda_0}\right)^2 \cdot 2 \neq 0$$

in which

$$t_1 = \alpha C_1, \quad t_2 = -(\alpha \lambda_0 C_3 + \beta C_4) \quad (3.13)$$

on the basis of literature^[9]. $F(u, \mu, \alpha, \beta)$ is a universal unfolding of $h(u, \mu)$.

By general singularity theory, the influences of parameters (α, β) on branch of solution for post-buckling are completely represented by the bifurcation diagram of the equation

$$F(u, \mu, \alpha, \beta) = 0 \quad (3.14)$$

Because $\langle Ce_1, e_1 \rangle \neq 0$, the Eq. (3.14) can be written as

$$F(u, \gamma, \alpha_1, \alpha_2) = u^3 + \alpha_1 u^2 - \gamma u + \alpha_2 = 0 \quad (3.15)$$

where

$$\alpha_1 = \frac{C_1}{C_0} \alpha = \frac{\langle Qe_1, e_1 \rangle}{\langle Ce_1, e_1 \rangle} \alpha$$

$$\left. \begin{aligned} \alpha_2 &= -\frac{\alpha\lambda_0 C_3 + \beta C_4}{C_0} = -\frac{\alpha\lambda_0 \langle p, e_1 \rangle + \beta \langle g, e_1 \rangle}{C_0} \\ \gamma &= \mu / \lambda_0 C_0 \end{aligned} \right\} \quad (3.16)$$

Because the universal unfolding includes all kinds of probable perturbations, different parameters decide the distinct structures of bifurcation diagram. We need even more to discuss the problem of persistence.

Let $F(\cdot, \cdot, \alpha)$ be a K -parameter unfolding of h . We say that $\alpha \in R^k$ is persistent if there is a neighborhood $U \subset R^k$ of α such that $\forall \beta \in U$, $F(\cdot, \cdot, \alpha)$ and $F(\cdot, \cdot, \beta)$ are contact equivalent (the definition of contact equivalent can be found in literature [9], otherwise say non-persistence). There are three kinds of non-persistence.

(a) Bifurcation point, the set is

$$\begin{aligned} F &= u^3 - \gamma u + \alpha_1 u^2 + \alpha_2 = 0 \\ F_u &= 3u^2 - \gamma + 2\alpha_1 u = 0 \\ F_{\gamma} &= -u = 0 \end{aligned}$$

Eliminating u and γ from the three equations, we have

$$\alpha_2 = 0 \quad (3.17)$$

(b) The set of hysteresis point is

$$\begin{aligned} F &= u^3 - \gamma u + \alpha_1 u^2 + \alpha_2 = 0 \\ F_u &= 3u^2 - \gamma + 2\alpha_1 u = 0 \\ F_{uu} &= 6u + 2\alpha_1 = 0 \end{aligned}$$

Eliminating u and γ , we get

$$\alpha_2 = \alpha_1^3 / 27 \quad (3.18)$$

(c) The set of double limit point is

$$\begin{aligned} F_u(u_1, \gamma, \alpha_1, \alpha_2) &= 3u_1^2 - \gamma + 2\alpha_1 u_1 = 0 \\ F_u(u_2, \gamma, \alpha_1, \alpha_2) &= 3u_2^2 - \gamma + 2\alpha_1 u_2 = 0 \\ F(u_1, \gamma, \alpha_1, \alpha_2) &= u_1^3 - \gamma u_1 + \alpha_1 u_1^2 + \alpha_2 = 0 \\ F(u_2, \gamma, \alpha_1, \alpha_2) &= u_2^3 - \gamma u_2 + \alpha_1 u_2^2 + \alpha_2 = 0 \\ u &\neq u_2 \end{aligned}$$

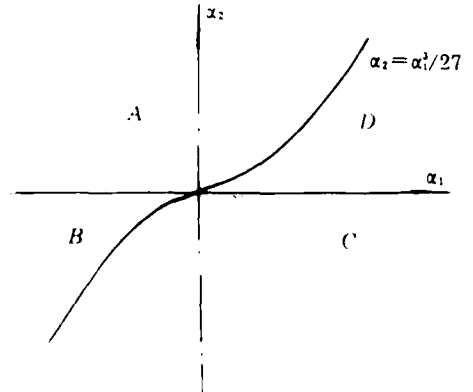


Fig. 1

We can easily obtain that the equations are no solutions. So we have

Theorem 3.3 $R^2 \setminus \{(\alpha_1, \alpha_2) \in R^2 \mid \alpha_2 = 0\} \cup \{(\alpha_1, \alpha_2) \in R^2 \mid \alpha_2 = \alpha_1^3 / 27\}$ divides a neighborhood of $(0, 0)$ into four connected components. When (α_1, α_2) and (α_1, α_2) are in the same connected component, $F(u, \gamma, \alpha_1, \alpha_2)$ and $F(u, \gamma, \alpha_1, \alpha_2)$ are contact equivalent. That is, they have the "same" bifurcation diagram.

The four connected components in a neighborhood of $(0, 0)$ are shown in Fig. 1.

We draw the bifurcation diagram in the four regions and on the two curves respectively. The bifurcation diagrams in region A (region C is the mirror image of region A), and in region D (region B is the mirror of region D) are shown in Fig. 2 and in Fig. 3.

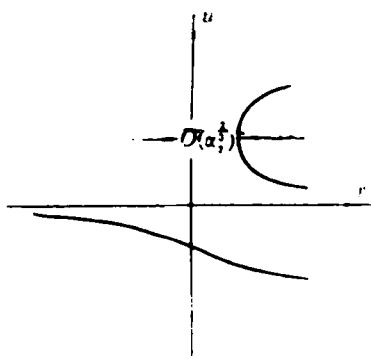


Fig. 2 ($\alpha_1 > 0$)

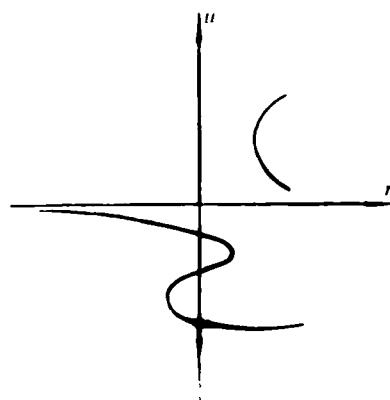


Fig. 3 ($\alpha_1 > 0, \alpha_2 > 0$)

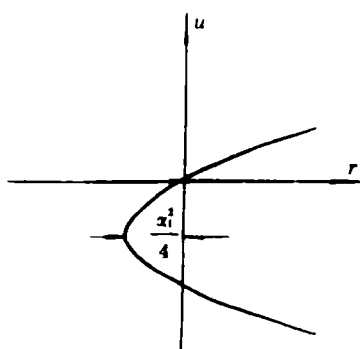


Fig. 4 ($\alpha_1 > 0$)

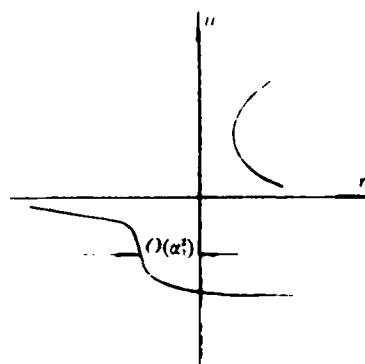


Fig. 5 ($\alpha_1 > 0$)

The diagram on $z=0$ is a pitchfork, according to the former discussion, the hysteresis curves can be get on $z_i = z_i^*$ 27. See Fig. 4 and Fig. 5.

By the results in this section, we now consider the influences on bifurcation solution in accordance with the parameters (z, β) which have distinct mechanics meaning. The following conclusions can be reached.

1. The deflection and transverse loading exist simultaneously at initial state. That is $z, \beta \neq 0$.

By (3.17) and (3.18), the two separating curves in $z-\beta$ plane are

$$\left. \begin{aligned} \alpha \lambda_0 \langle p, e_1 \rangle + \beta \langle g, e_1 \rangle &= 0 \\ -(\alpha \lambda_0 \langle p, e_1 \rangle + \beta \langle g, e_1 \rangle) &= \frac{\alpha^3}{27} \cdot \frac{C_1^3}{C_0^3} \end{aligned} \right\} \quad (3.19)$$

For fixed z and β , the bifurcation diagrams are determined by one of the Fig. 2 to Fig. 5.

2. There is only initial deflection imperfection, $z \neq 0, \beta = 0$. In this situation, the separating curves degenerate into

$$\alpha \langle p, e_1 \rangle = 0, \quad \alpha \lambda_0 \langle p, e_1 \rangle = -\frac{\alpha^3}{27} \cdot \frac{C_1^3}{C_0^3} \quad (3.20)$$

The universal unfolding is

$$F = u^3 - \gamma u + \frac{C_1}{C_0} \alpha u^2 - \frac{\alpha \lambda_0 \langle p, e_1 \rangle}{C_0} \quad (3.21)$$

1.° If $\langle p, e_1 \rangle = 0$, namely, $\langle w_0, e_1 \rangle = 0$, $C_1 = \langle Qe_1, e_1 \rangle \neq 0$, then the bifurcation diagram is shown Fig. 4. That the initial deflection imperfection parameter affects on the primitive critical loading is

$$\lambda^* = \lambda_0 - O(\alpha^2) \quad (3.22)$$

2.° If $w_0 = u_0 e_1$, ($u_0 \neq 0$), the mechanics meaning is that initial deflection is represented by characteristic function, then in view of properties of Q and C in Section II, we have

$$\begin{aligned} \left(\frac{C_1}{C_0} \alpha \right) \left(\frac{\alpha \lambda_0 \langle p, e_1 \rangle}{C_0} \right) &= \frac{\alpha^2}{C_0^2} \lambda_0 \langle Qe_1, e_1 \rangle \cdot \langle Aw_0, e_1 \rangle \\ &= \frac{\alpha^2}{C_0^2} u_0 \langle Qe_1, e_1 \rangle = \frac{3}{2} \frac{\alpha^2}{C_0^2} u_0^2 \|B(e_1, e_1)\|^2 \end{aligned}$$

Notice $\|B(e_1, e_1)\|^2 = \langle C(e_1), e_1 \rangle > 0$, so that the quadratic coefficient and the constant term in (3.21) are opposite sign. The bifurcation diagram is shown in Fig. 2. The influence on primitive critical loading affected by initial deflection imperfection parameter is a positive number, that is

$$\lambda^* = \lambda_0 + O(\alpha^{2/3}) \quad (3.23)$$

3.° If $\langle p, e_1 \rangle \neq 0$, namely, $\langle w_0, e_1 \rangle \neq 0$, $C_1 = \langle Qe_1, e_1 \rangle \neq 0$, then the first term in (3.20) is not satisfied and the pitchfork will not appear in an elastic plate. In addition, if α satisfies the second in Fig. 5, and the imperfection affected on primitive critical loading is

$$\lambda^* = \lambda_0 - O(\alpha^2) \quad (3.24)$$

3. There is no initial deflection, $\alpha = 0$, $\beta \neq 0$.

Now the two terms in (3.19) degenerate into

$$\beta \langle g, e_1 \rangle = 0 \quad (3.25)$$

The relevant universal unfolding is

$$F = u^3 - \gamma u - \frac{\beta \langle g, e_1 \rangle}{C_0} \quad (3.26)$$

Since there is no quadratic term in (3.26), the solving branch is shown in Fig. 2 when $\langle g, e_1 \rangle \neq 0$. That the influence on primitive critical loading affected by transverse loading parameter is the same as (3.23).

Finally, we should point out that this paper involves the buckling of a perfect elastic plate under the influence of small initial imperfections, first, as an asymptotic theory, its limitations are only applicable in a neighborhood of the bifurcation point, second, because the form of universal unfolding is determined by the nonlinear order of bifurcation equation, and Marguerre equation is an equation with the property of cubic nonlinearity (See Section II), the influences of two imperfection and transverse loading parameters at most to deal with the problem are decided.

Acknowledgment I am greatly indebted to Prof. Cheng Changjun and Prof. Zhou Zhiwei for their meticulous help in writing this paper.

References

- [1] J. M. T. Thompson and G. W. Hunt, Dangers of structural optimization, *Engineering Optimization*, **1** (1974), 99–110.
- [2] S. Timoshenko and S. Woinowsky-Krieger, *Theory of Plates and Shells*, McGraw-Hill, New York (1959).
- [3] J. M. Coan, The influence of curvature on the buckling of structures, *J. Appl. Mech.*, **18** (1951), 143.
- [4] K. Marguerre, Zur theorie gekrummten platte grosser formänderung, *Proc. 5th Intl. Cong. Appl. Mech.*, John Wiley & Sons Inc., New York (1938), 93–101.
- [5] Chia Chuenyuan, *Nonlinear Analysis of Plates*, McGraw-Hill, New York (1980), 93–107.
- [6] Xu Kaiyu, Buckling of a rectangular plate with small initial deflection, *ICNM-II Proc.*, ed. by Chien Weizang, Peking University Press, Beijing (1993) 630–632.
- [7] T. Poston and I. Stewart, *Catastrophe Theory and Its Applications*, Pitman, London (1978).
- [8] Z. Gaspar, Critical imperfection theory, *J. Struc. Mech.*, **11** (1983), 297–325.
- [9] M. Golubitsky and D. Schaeffer, A theory for imperfect bifurcation via singularity theory, *Comm. Pure Appl. Math.*, **32** (1979), 21–98.
- [10] M. Berger, On von Kármán equations and the buckling of a thin elastic plate I., The clamped plate, *Comm. Pure Appl. Mech.*, **20** (1967), 687–719.
- [11] Cheng Changjun and Zhu Zhengyou, *Buckling and Bifurcation of Structures*, Lanzhou University Press (1991). (in Chinese)
- [12] Zhang Gongqing and Lin Yuanqu, *Functional Analysis*, Peking University Press, Beijing (1987). (in Chinese)
- [13] Chen Yushu and Tang Yun, *Methods of Modern Analysis in Nonlinear Dynamics*, Science Press, Beijing (1992), 88–116. (in Chinese)