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THE CALCULATION OF EIGENVALUES FOR THE STATIONARY PERTURBATION OF COUETTE-POISEUILLE FLOW

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Abstract

The problem considered is that of two-dimensional viscous flow in a straight channel. The decay of a stationary perturbation from the Couette-Poiseuille flow in the downstream is sought. A differential eigenvalue equation resembling the Orr-Sommerfeld equation is solved by using a spectral method and an initial-value method (the compound matrix method) for values of the Reynolds number R between 0 and 2000. The eigenvalues are presented for several of interesting cases with different measures of mass flux. These eigenvalues determine the rate of decay for the purturbation.

Key words downstream, eigenvalue problem, Couette-Poiseuille flow

I. Introduction

Bramley and Dennis[1-3] and others[4, 5] obtained the eigenvalues of a stationary perturbation of Poiseuille flow for the viscous flow in a straight channel. In this paper we are concerned with the eigenvalue problem that governs the rate of decay for a stationary perturbation of Couette-Poiseuille flow. We also consider two-dimensional viscous motion in a straight channel. We assume that the difference between the base flow and Couette-Poiseuille flow decays exponentially downstream. It is then possible to seek solutions to the Navier-Stokes equations, far downstream, that are a perturbation to the Couette-Poiseulle profile and that decay exponentially in the downstream direction. The equations can then be linearized, yielding an ordinary differential eigenvalue system where the eigenvalues determine the rate of decay for the stationary perturbation.

To compute the eigenvalues for the stationary perturbation of Poseuille flow, several methods have been used. For example, a spectral method by Bramley[1] and Bramley and Dennis[2], an initial value method by Bramley and Dennis[3]. In this paper we compute the eigenvalues for the stationary perturbation of Couette-Poiseuille flow in a channel using these methods. These eigenvalues are important for the derivation of boundary conditions.

II. Equations

The channel width is taken to be h, and the kinematic viscosity ν. Then with length scale

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h, velocity scale U and Reynolds number $R=Uh/\nu$, the dimensionless streamfunction ψ satisfies the equation

$$R\left(\psi_y\frac{\partial}{\partial x}-\psi_x\frac{\partial}{\partial y}\right)\cdot\nabla^2\psi=\nabla^4\psi \tag{2.1}$$

where x is the (dimensionless) downstream coordinate and y is the (dimensionless) transverse coordinate. The origin is at the bottom of the channel. The flow far downstream approaches the Couette-Poiseuille flow and so

$$\psi\to\psi_0(y)=(3a-V)y^2+(V-2a)y^3$$
$$\psi_x\to 0$$

as $x\to\infty$. Where α is measure of mass flux and V is sliding velocity. $\psi_0(y)$ is streamfunction of Couette-Poiseuille flow. The motion of this flow is due to the relative displacement of the upper wall with respect to the lower one and a fall of pressure. Its velocity is not a symmetric function with respect to the central line. When $V=0$ and $\alpha=1$ it just is Poiseuille flow. For this case, it has been considered by Wilson[4] and others[1-3],[5]. So, we only consider the case of $V\neq 0$, and suppose $V=1$ for simplicity.

We now look for a perturbation solution where

$$\psi(x,y)=\psi_0(y)+\varepsilon\phi(y)e^{-\lambda x} \tag{2.2}$$

where

$$\psi_0(y)=(3a-1)y^2+(1-2a)y^3 \tag{2.3}$$

and ε is small. Substituting (2.2) in (2.1) and neglecting squares of ε leads to

$$\phi^{\mathrm{iv}}+2\lambda^2\phi''+\lambda^4\phi=\lambda R\{-\psi_0'(\phi''+\lambda^2\phi)+\psi_0'''\phi\}. \tag{2.4}$$

where

$$\psi_0'=2(3a-1)y+3(1-2a)y^2$$
$$\psi_0'''=6(1-2a)$$

with boundary conditions

$$\phi(0)=\phi(1)=\phi'(0)=\phi'(1)=0 \tag{2.5}$$

The above equation is similar to the Orr-Sommerfeld equation. The main difference is that in the present equation λ is an eigenvalue, not a prescribed wave-number; the equation is non-linear in λ, which in general will be complex. It can be shown that, if λ is an eigenvalue, so is λ^* (the complex conjugate). We are interested only in decaying modes, since growing modes cannot satisfy the boundary condition at $x=\infty$. Since there is symmetry about the real axis, as just noted, attention may be confined to the first quadrant of the λ plane. There is in fact an infinite sequence of eigenvalues here for each fixed R, which may be ordered by the magnitude of the real part.

The objective, then, is to calculate λ for all R. The corresponding eigenfunctions are of less interest. Physically, the most interesting problem is to find the eigenvalue with smallest real part, since this component of the disturbance persists longest.

The two methods are described in the next two sections. These methods are fully described in the relevant references and so only the main details are presented in this paper.

III. Compound Matrix Method

The compound matrix method was developed by Gilbert and Backus[6] and resurrected by Ng and Reid[7]. For the present it is convenient to rewrite Eq. (2.4) as a system of first-order equations. Thus, if we let $\Phi=[\phi,\phi', \phi'', \phi''']^T$, then Eq. (2.4) becomes

$$\Phi'=A\Phi \tag{3.1}$$

where

$$A=\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ c_2 & 0 & c_1 & 0 \end{pmatrix} \tag{3.2}$$

$$c_1=-2\lambda^2-\lambda R\psi_0'$$
$$c_2=\lambda R\psi_0'''-\lambda^3R\psi_0'-\lambda^4$$

Now let Φ_1 and Φ_2 be two solutions of Eq. (3.1) which satisfy the initial conditions

$$\Phi_1=[0,0,1,0]^T \text{ and } \Phi_2=[0,0,0,1]^T \tag{3.3}$$

and consider the 4×2 solution matrix

$$\Psi(y)=\begin{pmatrix} \phi_1 & \phi_2 \\ \phi_1' & \phi_2' \\ \phi_1'' & \phi_2'' \\ \phi_1''' & \phi_2''' \end{pmatrix} \tag{3.4}$$

The 2×2 minors of the matrix Ψ are

$$\left.\begin{array}{ll} p_1=\phi_1\phi_2'-\phi_1'\phi_2; & p_2=\phi_1\phi_2''-\phi_1''\phi_2 \\ p_3=\phi_1\phi_2'''-\phi_1'''\phi_2; & p_4=\phi_1'\phi_2''-\phi_1''\phi_2' \\ p_5=\phi_1'\phi_2'''-\phi_1'''\phi_2'; & p_6=\phi_1''\phi_2'''-\phi_1'''\phi_2'' \end{array}\right\} \tag{3.5}$$

Following Ng and Reid[7] it can be show that $P=(p_1,p_2,p_3,p_4,p_5,p_6)^T$ satisfies the system of differential equation

$$P'(y)=B(y)P(y) \tag{3.6}$$

where $B(y)$ is the 6×6 matrix given by

$$B(y)=\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & c_1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -c_2 & 0 & 0 & c_1 & 0 & 1 \\ 0 & -c_2 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{3.7}$$

and boundary condition

$$P(0)=[0,0,0,0,0,1]^T \tag{3.8}$$

The termination condition at $y=1$ is $p_1=0$. For given α and R, we choose an initial guss for λ and then modify it until $p_1(1)=0$.

IV. The Spectral Method

Orszag[8] discussed the advantages of Chebyshev polynomials relative to other sets of orthogonal polynomials. He showed that if the coefficients of a linear differential equation are infinitely differentiable, the approximation obtained is of infinite order in the sense that errors decrease more rapidly than any power of $1/N$ as $N\to\infty$, where N is the number of Chebyshev polynomials used in the approximation. We therefore use Chebyshev polynomials to obtain a numerical solution of (2.4), subject to boundary conditions (2.5).

The Chebyshev polynomials $T_n(z)$ are orthogonal over the interval $[-1, 1]$ with respect to the weight function $w(z)=(1-z^2)^{-1/2}$. Let us observe the formula $z=-1+2y$ maps $[0, 1]$ into $[-1, 1]$ and in the process derivatives with respect to y and z are related through the constant multiplying factor 2. It is convenient to assume from the outset that Eq. (2.4) and boundary conditions (2.5) are already formulated in the interval $[-1, 1]$. Suppose now that $\phi(y)=\phi((z+1))/2=\bar{\phi}(z)$ and the Chebyshev expansion of $\bar{\phi}(z)$ and its derivatives $d^q\bar{\phi}/dz^q$ be

$$\frac{d^q\bar{\phi}}{dz^q}=\sum_{n=0}^{\infty}a_n^{(q)}T_n(z) \tag{4.1}$$

where $a_n^{(0)}\equiv a_n$, and $T_n(z)$ is the nth degree Chebyshev polynomial of the first kind, defined by $T_n(\cos\theta)=\cos n\theta$, $n=0, 1, 2, \cdots$. The properties of $T_n(z)$ are now used to express $a_n^{(q)}$ in terms of a_n. Two constants c_n and d_n are commonly used in the recurrence relations of Chebyshev polynomials and are given as

$$\left.\begin{array}{ll} c_n=d_n=0 & (n<0);\quad c_0=2;\quad d_0=1 \\ c_n=d_n=1 & (n>0) \end{array}\right\} \tag{4.2}$$

It can be shown that[2]

$$c_na_n^{(2)}=\sum_{\substack{p=n+2\\ p\equiv n(\mathrm{mod}\,2)}}p(p^2-n^2)a_p \qquad (n\geqslant 0) \tag{4.3}$$

and

$$24c_na_n^{(4)}=\sum_{\substack{p=n+4\\ p\equiv n(\mathrm{mod}\,2)}}^{\infty}p(p^2-n^2)[(p-n)^2-4][(p+n)^2-4]a_p \quad (n\geqslant 0) \tag{4.4}$$

where $a\equiv b(\mathrm{mod}2)$ means that $a-b$ is divisible by 2. Using the properties $2zT_n(z)=T_{n+1}(z)+T_{n-1}(z)$ and $4z^2T_n(z)=T_{n-2}(z)+2T_n(z)+T_{n+2}(z)$, we can find that the nth Chebyshev coefficients of $2z\bar{\phi}(z)$ and $4z^2\bar{\phi}(z)$ are respectively

$$c_{n-1}a_{n-1}+a_{n+1} \qquad (n\geqslant 0) \tag{4.5}$$

and

$$c_{n-2}a_{n-2}+(c_n+c_{n-1})a_n+a_{n+2} \qquad (n\geqslant 0) \tag{4.6}$$

The similar results hold for $2z\bar{\phi}''(z)$ and $4z^2\bar{\phi}''(z)$ in terms of $a_n^{(2)}$ instead of a_n.

The above expansions are substituted into (2.4) and the coefficients of $T_n(z)$ may be equated. We shall now restrict the summation and truncate the Chebyshev series at $T_{N-1}(z)$. This gives

$$\begin{aligned}
&\frac{1}{24}\sum_{\substack{p=n+4\\ p=n(\mathrm{mod}\,2)}}^{N-1} p(p^2-n^2)[(p-n)^2-4][(p+n)^2-4]a_p\\
&+\sum_{\substack{p=n+2\\ p=n(\mathrm{mod}\,2)}}^{N-1}\left\{\left[\frac{1}{2}\lambda^2+\frac{1}{16}\lambda R(6a-1)+\frac{3}{64}\lambda R(1-2a)(c_n+c_{n-1})\right]\right.\\
&\cdot p(p^2-n^2)+\frac{3}{64}\lambda R(1-2a)[d_{n-2}p(p^2-(n-2)^2)+c_np(p^2\\
&\left.-(n+2)^2)]\right\}a_p\\
&+\frac{1}{16}\lambda R\sum_{\substack{p=n+1\\ p=n+1(\mathrm{mod}\,2)}}^{N-1}[c_np(p^2-(n+1)^2)+d_{n-1}p(p^2-(n-1)^2)]a_p\\
&+\frac{3}{16}\lambda R(1-2a)n(n-1)a_n-\frac{3}{8}\lambda R(1-2a)a_nc_n+\frac{1}{16}\lambda^4a_nc_n\\
&+\lambda^3R\left\{\frac{1}{64}[(6a-1)c_na_n+c_{n-1}a_{n-1}+c_na_{n+1}]\right.\\
&\left.+\frac{3}{256}(1-2a)[c_{n-2}a_{n-2}+(c_n^2+c_{n-1})a_n+c_na_{n+2}]\right\}\\
&=0, \qquad 0\leqslant n\leqslant N-1
\end{aligned} \tag{4.7}$$

for $0\leqslant n\leqslant N-1$. If the expansion (4.1) and the properties $T_n(\pm1)=(\pm1)^n$ and $T_n'(\pm1)=(\pm1)^{n-1}n^2$ are used on the boundary conditions $\phi(\pm1)=\phi'(\pm1)=0$, we obtain

$$\left.\begin{aligned}
&\sum_{n=0}^{N-1}a_n=0, \quad \sum_{n=0}^{N-1}n^2a_n=0\\
&\sum_{n=0}^{N-1}(-1)^na_n=0, \quad \sum_{n=0}^{N-1}(-1)^{n-1}n^2a_n=0
\end{aligned}\right\} \tag{4.8}$$

It is desirable to be able to calculate all the eigenvalues (real and complex) for a particular Reynolds number. Eq. (4.7) with $n=0, 1, \cdots, N-5$ together with the boundary conditions (4.8) can be expressed in the form

$$(\lambda^4C_4+\lambda^3C_3+\lambda^2C_2+\lambda C_1+C_0)D=0 \tag{4.9}$$

where C_i ($i=0, 1, 2, 3, 4$) are square matrices of order N. This can be transformed to the generalized eigenvalue problem

$$(E-\lambda F)X=0 \tag{4.10}$$

where

$$E=\begin{pmatrix} C_3 & C_2 & C_1 & C_0 \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{pmatrix}, \quad F=\begin{pmatrix} -C_4 & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{pmatrix}$$

and

$$X^T=(\lambda^3 D^T,\ \lambda^2 D^T,\ \lambda D^T,\ D^T)$$

The generalized eigenvalue problem given by (4.10) is solved using simple column operations and QR algorithm of Wilkinson[9].

V. Numerical Results

A large volume of numerical date is obtained and we try to present the main results in a way as concise as possible. The majority of these results are obtained by using the compound matrix method as stated in Section III. Because by using this method we can obtain accurate results. But, this method requires a reasonable initial approximation for λ. Finding an initial value for λ can be a problem when λ is complex because λ can be anywhere on the complex plane and therefore easily missed. With complex λ it is possible to use the spectral method to either calculate λ or at least obtain a good enough initial approximation for use with the initial value methods. The main results are given in following Tables 1−6. Real eigenvales λ may be presented in graphical from. Figs. 1−3, respectively, give the positive eigenvalues for $\alpha=0$, 0.5 and 1. Fig. 4 give the first several positive eigenvalues for $\alpha=10$. It will be seen that in both cases there are values of R below which there are no positive real eigenvalues. These values (called R_c) are presented in Tables 1−6, respectively.

Table 1 The first three branches eigenvalues with positive real part for $0\leqslant R\leqslant 2000$ and $\alpha=0$

R	λ_I	λ_{II}	λ_{III}
0	$4.21239+2.25073i$	$7.49768+2.76868i$	$10.71254+3.10315i$
0.25	$4.22380+2.3353i$	$7.50326+2.76386i$	$10.71573+3.10109i$
0.5	$4.23519+2.21608i$	$7.50885+2.75895i$	$10.71895+3.09893i$
1.0	$4.25787+2.18044i$	$7.52005+2.74883i$	$10.72546+3.09434i$
1.5	$4.28044+2.14378i$	$7.53130+2.73828i$	$10.73208+3.08937i$
2.5	$4.32517+2.06726i$	$7.55391+2.71575i$	$10.74565+3.07824i$
5.0	$4.43440+1.85533i$	$7.61108+2.64913i$	$10.78199+3.04328i$
10	$4.64290+1.30467i$	$7.72355+2.44715i$	$10.86985+2.94104i$
25	3.22247:9.80471	$6.68234+1.49752i$	$11.29275+2.29286i$
50	2.28037:11.72536	$5.65550+1.39696i$	$11.51583+1.77596i$
100	1.48560:10.07397	4.08188:4.44360	$10.53017+2.45074i$
250	0.70176	1.51283:3.57906	$7.55767+1.88303i$
500	0.36374	0.75341:2.40803	$4.85975+1.03113i$
1000	0.18368	0.37621:1.44507	2.55022:3.27058
2000	0.09208	0.18804:0.78598	1.18945:2.06525
R_c	14.607	98.764	850.254
λ_c	4.84950	4.28484	3.30495

Table 2 The next three branches eigenvalues with positive real part for $0 \leqslant R \leqslant 100$ and $\alpha = 0$

R	λ_{I}	λ_{II}	λ_{III}
0	13.89996+3.35221i	17.07336+3.55109i	20.23852+3.71677i
0.25	13.90201+3.35112i	17.07480+3.55044i	20.23957+3.71635i
0.5	13.90410+3.34995i	17.07626+3.54971i	20.24066+3.71585i
1.0	13.90838+3.34735i	17.07929+3.54802i	20.24293+3.71464i
1.5	13.91278+3.34442i	17.08247+3.54605i	20.24535+3.71318i
2.5	13.92202+3.33754i	17.08925+3.54119i	20.25059+3.70945i
5.0	13.94771+3.31450i	17.10878+3.52398i	20.26615+3.69552i
10	14.01158+3.24348i	17.15942+3.46823i	20.30779+3.64857i
25	14.31544+2.80139i	17.40443+3.11019i	20.51837+3.34105i
50	16.41647+2.39431i	18.64087+2.35802i	21.45211+2.25358i
100	17.11561+2.66385i	16.71284:31.76841	22.79124+2.67424i
R_c		75.069	
λ_c		20.68461	

Table 3 The first three branches eigenvalues with positive real part for $0 \leqslant R \leqslant 2000$ and $\alpha = 0.5$

R	λ_{I}	λ_{II}	λ_{III}
0	4.21239+2.25073i	7.49768+2.76868i	10.71254+3.10315i
0.25	4.18133+2.25046i	7.46657+2.76844i	10.68140+3.10293i
0.5	4.15063+2.24967i	7.43574+2.76772i	10.65049+3.10229i
1.0	4.09037+2.24659i	7.37493+2.76493i	10.58933+3.09975i
1.5	4.03161+2.24161i	7.31524+2.76040i	10.52906+3.09561i
2.5	3.91864+2.22643i	7.19924+2.74656i	10.41119+3.08282i
5.0	3.66284+2.16334i	6.92856+2.68799i	10.13167+3.02719i
10	3.25798+1.96956i	6.46417+2.49559i	9.63396+2.83411i
25	2.59468+1.27601i	5.49462+1.64631i	8.48659+1.90593i
50	1.95130+0.27690i	4.10187+1.08876i	6.87312+1.69622i
100	1.32497:0.8246	2.99759+0.92741i	5.25462+0.92413i
250	0.55342:0.32225	1.73936:1.40179	2.98547+0.64521i
500	0.27822:0.16066	0.90988:0.67717	1.83556:1.59553
1000	0.13930:0.08027	0.45849:0.3364	0.95164:0.7734
2000	0.06972:0.04013	0.22966:0.16807	0.47848:0.38484
R_c	55.173	196.29	417.32
λ_c	1.8245	1.97037	2.0424

As in [2], [3], [5], we present several tables that give the computed eigenvalues. Tables 1, 3, 5 and 6 give the first three branches eigenvalues with positive real part and $0 \leqslant R \leqslant 2000$ for $\alpha = 0$, 0.5, 1 and 10, respectively. The next three branches eigenvalues with positive real part for $\alpha = 0$ and 0.5 are given in Tables 2 and 4, respectively. The complex eigenvalues occur in complex conjugate pairs but only the one with positive imaginary part is given.

From above tables and figures, we can conclude that each branch of eigenvalue is complex as $0 \leqslant R < R_c$ and its conjugate also being an eigenvalue. At $R = R_c = R_{c1}$ each branch of complex solutions and its conjugate coalesce on the real axis and, then for R larger then R_c

they split into two branches of real solutions. These two real solutions may be coalesce at $R=R_{c2}$. Then, for larger value of R these two solutions split into a complex solution and its conjugate, and again they remain complex for $R_{c2}<R<R_{c3}$. Then at $R=R_{c3}$ the complex solution and its conjugate again coalesce on the real axis, and the cycle also may be repeated. For example, for the second branch eigenvalus of $\gamma=1$ (see Table 5), $R_{c2}=25.93$, $\lambda_{c2}=3.60473$ and $R_{c3}=126.375$, $\lambda_{c3}=1.62096$. But in each of cases, for large values of R, we have not present the eigenvalues that have large modulus. This is because these eigenvalues hard to be exactly obtained.

Table 4 The next three branches eigenvalues with positive real part for $0\leq R\leq 2000$ and $\gamma=0.5$

R	λ_{I}	λ_{II}	λ_{III}
0	13.89996+3.35221i	17.07336+3.55109i	20.23852+3.71677i
0.25	13.86880+3.35201i	17.04219+3.55089i	20.20733+3.71658i
0.5	13.83783+3.35140i	17.0118+3.55031i	20.17628+3.71601i
1.0	13.77643+3.34901i	16.94961+3.54801i	20.11460+3.71377i
1.5	13.71577+3.34510i	16.88866+3.54423i	20.05344+3.71007i
2.5	13.59663+3.3329i	16.76862+3.53238i	19.93272+3.69844i
5.0	13.31125+3.27893i	16.47909+3.47937i	19.64011+3.64599i
10	12.79155+3.08485i	15.94374+3.28406i	19.09307+3.44939i
25	11.52371+2.11583i	14.58882+2.29525i	17.67265+2.45156i
50	9.76966+2.04289i	12.71889+2.27322i	15.70259+2.44090i
100	7.49648+1.24868i	10.10028+1.68206i	12.85479+1.94463i
250	4.45834+0.51014i	6.37132+1.04679i	8.47196+0.95776i
500	2.84549+0.41171i	3.77060+0.44898i	5.41394+0.48462i
1000	1.60382:1.40363	2.31280+0.09734i	3.09662+0.40281i
2000	0.81488:0.69156	1.23620:1.09066	1.73423:1.58971
R_c	716.39	1092.35	1544.37
λ_c	2.08841	2.12184	2.14902

Table 5 The first three branches eigenvalues with positive real part for $0\leq R\leq 2000$ and $\gamma=1$

R	λ_{I}	λ_{II}	λ_{III}
0	4.21239+2.25073i	7.49768+2.76868i	10.71254+3.10315i
0.25	4.13953+2.26605i	7.43025+2.77238i	10.64731+3.10423i
0.5	4.06879+2.27806i	7.36413+2.77405i	10.58301+3.10349i
1.0	3.93363+2.29318i	7.23577+2.77167i	10.45716+3.09684i
1.5	3.80680+2.29807i	7.11250+2.76234i	10.33496+3.08372i
2.5	3.57733+2.28364i	6.8809 +2.72553i	10.10138+3.03978i
5.0	3.12956+2.16151i	6.38394+2.55160i	9.57884+2.84117i
10	2.60931+1.80932i	5.68659+1.98122i	8.77758+2.11704i
25	2.13872+0.85471i	3.99606:3.50936	6.56289+1.49964i
50	1.44953:1.03477	2.56145+1.03812i	5.22028+0.61330i
100	0.74513:0.49014	2.04604+0.37869i	2.72817+0.52722i
250	0.29972:0.19375	0.90994:0.70903	1.72534+0.10246i
500	0.14997:0.09672	0.45773:0.34951	0.92598:0.77532
1000	0.07500:0.04834	0.22916:0.17420	0.46552:0.38307
2000	0.03750:0.02417	0.11462:0.08703	0.23302:0.19104
R_c	36.046	18.755	272.482
λ_c	1.73512	4.81965	1.57878

Table 6 The first three branches eigenvalues with positive real part for $0 \leqslant R \leqslant 2000$ and $\alpha=10$

R	λ_{I}	λ_{II}	λ_{III}
0	4.21239+2.25073i	7.49768+2.76868i	10.71254+3.10315i
0.25	3.49891+2.3803i	6.83919+2.75413i	10.07626+3.04631i
0.5	3.00962+2.30292i	6.31629+2.58960i	9.53695+2.83658i
1.0	2.45788+2.01131i	5.61056+2.01197i	8.7417+2.02122i
1.5	2.19221+1.73831i	5.27161+1.10079i	9.25562:7.19156
2.5	2.0007 +1.31996i	4.25990:3.00621	4.77162:4.2599
5.0	2.12834+1.16566i	1.50206:1.28753	6.0948:7.30865
10	1.79663+0.99479i	0.75407:0.58234	2.31387:2.8449
25	1.63573+0.8150i	0.30199:0.22826	0.80192:0.92176
50	1.51219+0.70873i	0.15102:0.1138	0.38829:0.46098
100	1.39278+0.62013i	0.07552:0.05686	0.19279:0.2305
250	1.25286+0.51819i	0.03021:0.02274	0.07697:0.0922
500	1.16037+0.45272i	0.01510:0.01137	0.03847:0.0461
1000	0.99256+0.00669i	0.00755:0.00568	0.01923:0.02305
2000	0.96206+0.00564i	0.00377:0.00284	0.00966:0.01153
R_c		1.697	1.386
λ_c		5.3756	8.52505

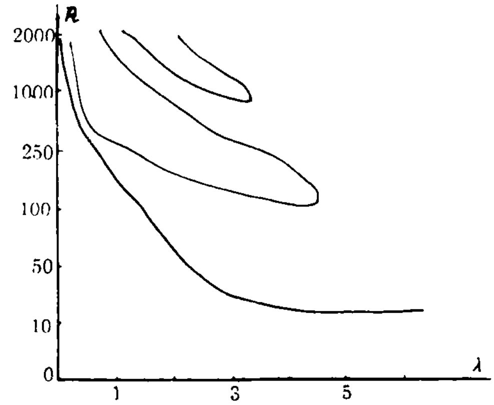

Fig. 1 Graph of positive real eigenvalues against Reynolds number for $\alpha=0$

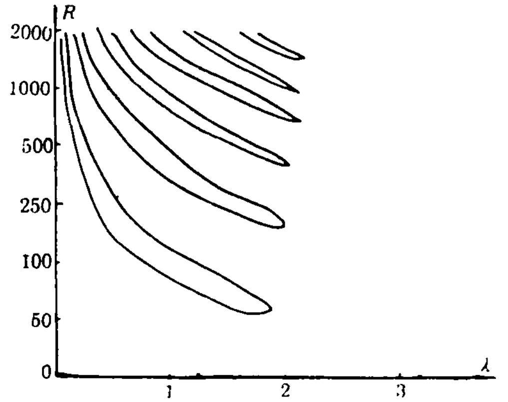

Fig. 2 Graph of positive real eigenvalues against Reynolds number for $\alpha=0.5$

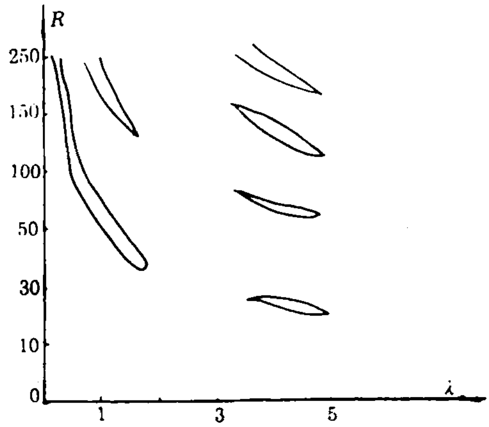

Fig. 3 Graph of positive real eigenvalues against Reynolds number for $\alpha=1$

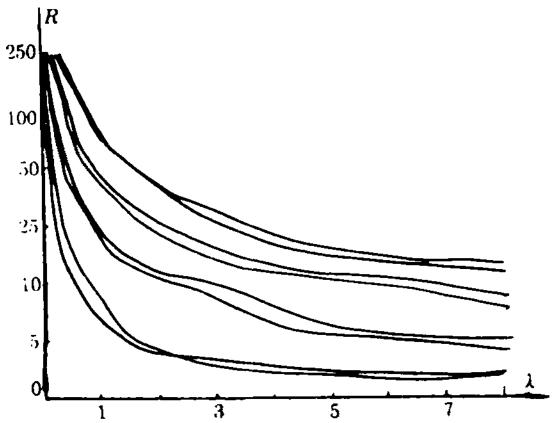

Fig. 4 Graph of the first several positive real eigenvalues against Reynolds number for $\alpha=10$

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References

[1] J. S. Bramely, Note on the calculation of eigenvalues for stationary perturbation of Poiseuille flow, *J. Comput. Phys.*, **53** (1984), 524—529.

[2] J. S. Bramley and S. C. R. Dennis, The calculation of eigenvalues for the stationary perturbation of Poiseuille flow, *J. Comput. Phy.*, **47** (1982), 179—198.

[3] J. S. Bramley and S. C. R. Dennis, The calculation of eigenvalues for the stationary perturbation of Poiseuille flow using initial value methods, *J. Math. Anal. Appl.*, **101** (1984), 30—38.

[4] S. D. R. Wilson, The development of Poiseuille flow, *J. Fluid Mech.*, **38** (1969), 793—806.

[5] G. A. Ache, Computation of the eigenvalues for perturbations of Poiseuille flow using a two-point boundary value method, *SIAM J. Sci. Statist. Comput.*, **10** (1989), 1097—1112.

[6] F. Gilbert and G. E. Backus, Propagator matrices in elastic wave and vibration problems, *Geophysics*, **31** (1966), 326—332.

[7] B. S. Ng and W. H. Reid, An initial value method for eigenvalue problems using compound matrices, *J. Comput. Phys.*, **30** (1979), 125—136.

[8] S. A. Orszag, Accurate solution to the Orr-Sommerfeld stability equation, *J. Fluid Mech.*, **50** (1971), 689—703.

[9] J. H. Wilkinson, *The Algebraic Eigenvalue Problem*, Clarendon, Oxford (1965).