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PLASTIC BUCKLING OF STIFFENED TORISPHERICAL SHELL

Hao Gang (郝　刚)[1]　Zeng Guangwu (曾广武)[1]　Hao Qiang (郝　强)[1]

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Abstract

This paper uses the nonlinear prebuckling consisten theory to analyse the plastic buckling problem of stiffened torispherical shell under uniform external pressure. The buckling equation and energy expressions of the shell are built, the calculation formula is presented. Numerical examples show that the method in this paper has better precision and the calculating process is very simple.

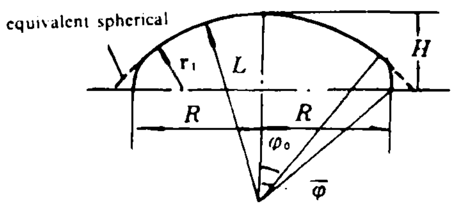
Key words stiffened shell of revolution, torispherical shell, uniform external pressure, plastic buckling

I. Introduction

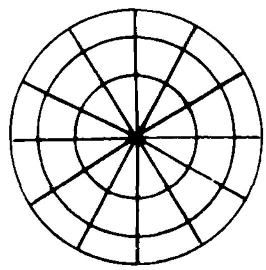
Stiffened torispherical shell is a shell of revolution, which consists of spherical shell and a hoop shell, it is often used in engineering as a head of vessels and submarine, and component of missiles. The shell must bear uniform external pressure and buckling is one of the main forms of collapse. In engineering the critical pressure is often estimated by the formula of stiffened spherical shell and it shall produce some errors obviously. It shall take a lot of calculation expenses and times and it is not suitable in the primary design. This paper uses the nonlinear prebuckling consistent theory to analyse the plastic buckling of this kind of shell, the calculation formula is presented on the energy principle, the method and formula can solve the buckling problems of stiffened spherical shell and stiffened torispherical shell.

II. Basic Equations

The basic form of torispherical shell disscussed in the paper is shown in Fig. 1.



(a)



(b)

Fig. 1. The basic form of the shell

[1] Dept. of Naval Architecture and Ocean Engineering, Huazhong University of Science and Technology, Wuhan 430074, P. R. China

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The strain relations of shell can be written from Ref. [1]:

$$\left.\begin{aligned}
\varepsilon_{\varphi}&=\varepsilon_{\varphi}^{(1)}+\varepsilon_{\varphi}^{(2)}=\left\{u'+\frac{w}{R_1}\right\}+\{0.5(\beta^2+\gamma^2)\}\\
\varepsilon_{\theta}&=\varepsilon_{\theta}^{(1)}+\varepsilon_{\theta}^{(2)}=\left\{\frac{\dot{v}}{r}+ur'/r+\frac{w}{R_2}\right\}+\left\{\frac{1}{2}(\psi^2+\gamma^2)\right\}\\
\varepsilon_{\varphi\theta}&=\varepsilon_{\varphi\theta}^{(1)}+\varepsilon_{\varphi\theta}^{(2)}=\left\{\frac{\dot{u}}{r}+r\left(\frac{v}{r}\right)'\right\}+\{\beta\psi\}\\
k_{\varphi}&=\beta',\quad k_{\theta}=\frac{\dot{\psi}}{r}+\beta\frac{r'}{r},\quad k_{\varphi\theta}=\frac{\dot{\beta}}{r}-\psi\frac{r'}{r}-\frac{r'}{R_2}
\end{aligned}\right\}\tag{2.1}$$

where,

$$r=R_2\sin\varphi,\quad \beta=\frac{u}{R_1}-w',\quad \psi=\frac{v}{R_2}-\frac{\dot{w}}{r},\quad \gamma=\frac{1}{2}\left(-\frac{\dot{u}}{r}+v'+\frac{vr'}{r}\right),$$

$$(\ )'=\frac{\partial(\ )}{R_1\partial\varphi},\quad (\dot{\ })=\frac{\partial(\ )}{\partial\theta}.$$

Superscript (1) and (2) are respectively expressed as linear and nonlinear part in the strain. R_1 and R_2 are respectively meridional and circumferential of curvature.

Assuming prebuckling state is "e" and buckling state is "p", the strain relations before buckling can be obtained by taking prebuckling displacement pattern into formula (2.1) and the strain relation can be shown as when buckling:

$$\left.\begin{aligned}
\varepsilon_{p\varphi}&=\varepsilon_{p\varphi}^{(1)}+\varepsilon_{p\varphi}^{(2)}+\varepsilon_{ep\varphi}=\left\{u_p'+\frac{w_p}{R_1}\right\}+\left\{\frac{1}{2}(\beta_p^2+\gamma_p^2)\right\}+\{\beta\beta_p+\gamma\gamma_p\}\\
\varepsilon_{p\theta}&=\varepsilon_{p\theta}^{(1)}+\varepsilon_{p\theta}^{(2)}+\varepsilon_{ep\theta}=\left\{\dot{v}_p\frac{1}{r}+\frac{u_pr'}{r}+\frac{w_p}{R_2}\right\}+\left\{\frac{1}{2}(\psi_p^2+\gamma_p^2)\right\}+\{\psi\psi_p+\gamma\gamma_p\}\\
\varepsilon_{p\varphi\theta}&=\varepsilon_{p\varphi\theta}^{(1)}+\varepsilon_{p\varphi\theta}^{(2)}+\varepsilon_{ep\varphi\theta}=\left\{\frac{\dot{u}_p}{r}+r\left(\frac{v_p}{r}\right)'\right\}+\{\beta_p\psi_p\}+\{\beta\psi_p+\psi\beta_p\}\\
k_{p\varphi}&=\beta_p',\quad k_{p\theta}=\frac{\dot{\psi}_p}{r}+\beta_p\frac{r'}{r},\quad k_{p\varphi\theta}=\frac{\dot{\beta}_p}{r}-\psi_p\frac{r'}{r}-\frac{v_p'}{R_2}
\end{aligned}\right\}\tag{2.2}$$

where

$$\beta_p=\frac{u_p}{R_1}-w_p',\quad \psi_p=\frac{v_p}{R_2}-\frac{\dot{w}_p}{r},\quad \gamma_p=\frac{1}{2}\left(-\frac{\dot{u}_p}{r}+v_p'+\frac{v_pr'}{r}\right)$$

Subscript "ep" expresses the strain consisting of prebuckling and buckling deformation. Assuming θ_m is the circumferential coordinate of the m-th meridional stiffener, Z_m is the distance of centriod of the stiffener area from shell midsurface, leting $\theta=\theta_m$ and substituting Eq. (2.1) and (2.2), we can obtain the strian relations of the m-th meridional stiffener:

$$\bar{\varepsilon}_{m\varphi}=\varepsilon_{m\varphi}+Z_mk_{m\varphi},\quad \bar{\varepsilon}_{pm\varphi}=\varepsilon_{pm\varphi}+Z_mk_{pm\varphi}\tag{2.3}$$

where,

$$\varepsilon_{m\varphi}=\varepsilon_{\varphi}|_{\theta=\theta_m},\quad \varepsilon_{pm\varphi}=\varepsilon_{p\varphi}|_{\theta=\theta_m},\quad k_{m\varphi}=k_{\varphi}|_{\theta=\theta_m},\quad k_{pm\varphi}=k_{p\varphi}|_{\theta=\theta_m}$$

In a similar manner, the strain relations of c-th circumferential stiffener:

$$\bar{\varepsilon}_{c\theta}=\varepsilon_{c\theta}+Z_c k_{c\theta},\quad \bar{\varepsilon}_{\varphi c\theta}=\varepsilon_{\varphi c\theta}+Z_c k_{\varphi c\theta} \tag{2.4}$$

where,

$$\varepsilon_{c\theta}=\varepsilon_\theta|_{\varphi=\varphi_c},\quad \varepsilon_{\varphi c\theta}=\varepsilon_{\varphi\theta}|_{\varphi=\varphi_c},\quad k_{c\theta}=k_\theta|_{\varphi=\varphi_c},\quad k_{\varphi c\theta}=k_{\varphi\theta}|_{\varphi=\varphi_c}$$

φ_c is the meridional coordinate of c-th circumferential stiffener, Z_c is the distance of centroid of the stiffener area from shell midsurface. Taking plastic buckling theory and Mises yield principle, the relations between stress and strain in the shell can be expressed based on plastic shape theory[2]:

$$\left.\begin{aligned}
d\sigma_\varphi&=\frac{E}{1-\mu^2}(a_{11}d\varepsilon_\varphi+\mu a_{12}d\varepsilon_\theta+a_{13}d\varepsilon_{\varphi\theta})\\
d\sigma_\theta&=\frac{E}{1-\mu^2}(\mu a_{12}d\varepsilon_\varphi+a_{22}d\varepsilon_\theta+a_{23}d\varepsilon_{\varphi\theta})\\
d\tau_{\varphi\theta}&=\frac{E}{1-\mu^2}\left(a_{13}d\varepsilon_\varphi+a_{23}d\varepsilon_\theta+\frac{1-\mu}{2}a_{33}d\varepsilon_{\varphi\theta}\right)
\end{aligned}\right\} \tag{2.5}$$

where,

$$\left.\begin{aligned}
a_{11}&=\frac{4(1-\mu^2)\phi_s}{3[1-(1-2\mu)\phi_s/3]}\left\{\frac{1}{1+(1-2\mu)\phi_s}-\frac{3(1-\phi_t/\phi_s)}{4[1-(1-2\mu)\phi_s/3]}\cdot\frac{K_1^2}{K}\right\}\\
a_{22}&=\frac{4(1-\mu^2)\phi_s}{3[1-(1-2\mu)\phi_s/3]}\left\{\frac{1}{1+(1-2\mu)\phi_s}-\frac{3(1-\phi_t/\phi_s)}{4[1-(1-2\mu)\phi_s/3]}\cdot\frac{K_2^2}{K}\right\}\\
a_{33}&=\frac{2(1+\mu)\phi_s}{[1-(1-2\mu)\phi_s/3]}\cdot\left\{\frac{1}{3}-\frac{(1-\phi_t/\phi_s)}{[1-(1-2\mu)\phi_s/3]}\cdot\left(\frac{\tau_{\varphi\theta}}{\sigma_i}\right)^2\right\}\\
a_{12}&=\frac{2(1-\mu^2)\phi_s}{3\mu[1-(1-2\mu)\phi_s/3]}\cdot\left\{\frac{1-(1-2\mu)\phi_s}{1+(1-2\mu)\phi_s}-\frac{3(1-\phi_t/\phi_s)}{2[1-(1-2\mu)\phi_s/3]}\cdot\frac{K_1K_2}{K}\right\}\\
a_{13}&=\frac{(1-\mu^2)(\phi_s-\phi_t)}{[1-(1-2\mu)\phi_s/3][1-(1-2\mu)\phi_t/3]}\cdot\frac{\tau_{\varphi\theta}}{\sigma_i}\cdot\frac{K_1}{K}\\
a_{23}&=\frac{(1-\mu^2)(\phi_s-\phi_t)}{[1-(1-2\mu)\phi_s/3][1-(1-2\mu)\phi_t/3]}\cdot\frac{\tau_{\varphi\theta}}{\sigma_i}\cdot\frac{K_2}{K}
\end{aligned}\right\} \tag{2.6}$$

$$\left.\begin{aligned}
K&=1-\frac{(1-2\mu)(\phi_s-\phi_t)(\sigma_\varphi+\sigma_\theta)^2}{[1+(1-2\mu)\phi_s][3-(1-2\mu)\phi_t]\sigma_i^2}\\
K_1&=\frac{1}{\sigma_i}\cdot\left\{\frac{1+(1-2\mu)\phi_s/3}{1+(1-2\mu)\phi_s}\sigma_\varphi-\frac{2(1-2\mu)\phi_s/3}{1+(1-2\mu)\phi_s}\sigma_\theta\right\}\\
K_2&=\frac{1}{\sigma_i}\cdot\left\{-\frac{2(1-2\mu)\phi_s/3}{1+(1-2\mu)\phi_s}\sigma_\varphi+\frac{1+(1-2\mu)\phi_s/3}{1+(1-2\mu)\phi_s}\sigma_\theta\right\}\\
\phi_s&=E_s^0/E,\quad \phi_t=E_t^0/E
\end{aligned}\right\} \tag{2.7}$$

where, E_s^0 and E_t^0 are respectively secant and tangent module in the figure of single stress and strain, the relations between the moduli and E_s and E_t, which are respectively secant and tangent module in the figure of the stress strength and strain strength, can be shown as:

$$\left.\begin{aligned}\frac{1}{E_s^0}=\frac{1-2\mu}{3E}+\frac{1}{E_s},\quad E_s=\frac{\sigma_i}{e_i}\\ \frac{1}{E_t^0}=\frac{1-2\mu}{3E}+\frac{1}{E_t},\quad E_t=\frac{d\sigma_i}{de_i}\end{aligned}\right\}\tag{2.8}$$

The corresponding yield strength is:

$$\left.\begin{aligned}&\sigma_i^2=\sigma_\varphi^2+\sigma_\theta^2-\sigma_\varphi\sigma_\theta+3\tau_{\varphi\theta}^2\\ &e_i^2=\frac{1}{3}\left\{(e_\varphi+e_\theta)^2+\varepsilon_{\varphi\theta}^2+\frac{3(e_\varphi+e_\theta)^2}{[1+4(1-2\mu)E_s/(3E)]^2}\right\}\end{aligned}\right\}\tag{2.9}$$

Ignoring the effects of the trauerse strain and torque, the stiffener is in the single stress state. For the meridional stiffener:

$$d\bar{\sigma}_\varphi=a_m E d\bar{e}_\varphi\tag{2.10}$$

where

$$\left.\begin{aligned}&a_m=\frac{4\phi_s}{[3(1-\phi_s/3)]}\cdot\left\{\frac{1}{(1+\phi_s)}-\frac{3}{4}\cdot\frac{(1-\phi_s/\phi_t)}{(1-\phi_s/3)}\cdot\frac{K_1^2}{K}\right\}\\ &K_1=\frac{(1+\phi_s/3)}{1+\phi_s},\quad K=1-\frac{\phi_s-\phi_t}{(1+\phi_s)(3-\phi_t)}\end{aligned}\right\}\tag{2.11}$$

For the circumferential stiffener:

$$d\bar{\sigma}_\theta=a_0 E d\bar{e}_\theta\tag{2.12}$$

where,

$$\left.\begin{aligned}&a_0=\frac{4\phi_s}{[3(1-\phi_s/3)]}\cdot\left\{\frac{1}{(1+\phi_s)}-\frac{3}{4}\cdot\frac{(1-\phi_s/\phi_t)}{(1-\phi_s/3)}\cdot\frac{K_2^2}{K}\right\}\\ &K_2=\frac{(1+\phi_s/3)}{1+\phi_s}\end{aligned}\right\}\tag{2.13}$$

III. Energy Expression

The total energy of stiffened torispherical shell can be expressed as:

$$\Pi=V_s+V_\theta+V_\varphi+U\tag{3.1}$$

where, V_s, V_θ and V_φ are respectively strain energy of shell, meridional and circumferential stiffener, U is potential energy.

Every part of energy can be resolved according to the state and type of strain, the strain can be expressed in following in this paper:

$$e=e_s^{(1)}+e_s^{(2)}+e_\varphi^{(1)}+e_\varphi^{(2)}+e_{s\varphi},\quad k=k_s+k_\varphi\tag{3.2}$$

where, $e_s^{(i)}$ and $e_\varphi^{(i)}$ ($i=1,2$) are linear and nonlinear item of prebuckling and buckling strain respectively, $e_{s\varphi}$ is the strain consisting of two kinds of strains.

The expression of every part strain energy is given in following:

3.1 Strain energy of the shell

$$V_{si}^{(1)}=\frac{C}{2}\int_V\left\{\left[a_{11}(e_{i\varphi}^{(1)})^2+a_{22}(e_{i\theta}^{(1)})^2+2\mu a_{12}e_{i\varphi}^{(1)}e_{i\theta}^{(1)}+\frac{1-\mu}{2}a_{33}(e_{i\varphi\theta}^{(1)})^2\right]\right.$$

$$+Z^2[a_{11}k_{i\varphi}^2+a_{22}k_{i\theta}^2+2\mu a_{12}k_{i\varphi}k_{i\theta}+2(1-\mu)a_{33}k_{i\varphi\theta}^2]\Big\}dv$$

$$V_{si}^{(2)}=\frac{C}{2}\int_V\Big[a_{11}(\varepsilon_{i\varphi}^{(2)})^2+a_{22}(\varepsilon_{i\theta}^{(2)})^2+2\mu a_{12}\varepsilon_{i\varphi}^{(2)}\varepsilon_{i\theta}^{(2)}+\frac{1-\mu}{2}a_{33}(\varepsilon_{i\varphi\theta}^{(2)})^2\Big]dv$$

$$V_{si}^{(12)}=C\int_V\Big[a_{11}\varepsilon_{i\varphi}^{(1)}\varepsilon_{i\varphi}^{(2)}+a_{22}\varepsilon_{i\theta}^{(1)}\varepsilon_{i\theta}^{(2)}+\mu a_{12}(\varepsilon_{i\varphi}^{(1)}\varepsilon_{i\theta}^{(2)}+\varepsilon_{i\varphi}^{(2)}\varepsilon_{i\theta}^{(1)})$$

$$+\frac{1-\mu}{2}a_{33}\varepsilon_{i\varphi\theta}^{(1)}\varepsilon_{i\varphi\theta}^{(2)}\Big]dv$$

$$V_{sep}^{(1)}=C\int_V\Big\{\Big[a_{11}\varepsilon_{e\varphi}(\varepsilon_{p\varphi}^{(1)}+\varepsilon_{ep\varphi}^{(1)})+a_{22}\varepsilon_{e\theta}(\varepsilon_{p\theta}^{(1)}+\varepsilon_{ep\theta}^{(1)})$$

$$+\mu a_{12}(\varepsilon_{e\varphi}\varepsilon_{p\theta}^{(1)}+\varepsilon_{e\theta}\varepsilon_{p\varphi}^{(1)}+\varepsilon_{e\varphi}\varepsilon_{ep\theta}^{(1)}+\varepsilon_{e\theta}\varepsilon_{ep\varphi}^{(1)})+\frac{1-\mu}{2}a_{33}\varepsilon_{e\varphi\theta}(\varepsilon_{p\varphi\theta}^{(1)}+\varepsilon_{ep\varphi\theta}^{(1)})\Big]$$

$$+Z^2[a_{11}k_{e\varphi}k_{p\varphi}+a_{22}k_{e\theta}k_{p\theta}+\mu a_{12}(k_{e\varphi}k_{p\theta}+k_{e\theta}k_{p\varphi})$$

$$+2(1-\mu)a_{33}k_{e\varphi\theta}k_{p\varphi\theta}]\Big\}dv$$

$$V_{sep}^{(2)}=\frac{C}{2}\int_V\Big[a_{11}(\varepsilon_{ep\varphi}^{(1)})^2+a_{22}(\varepsilon_{ep\theta}^{(1)})^2+2\mu a_{12}\varepsilon_{ep\varphi}^{(1)}\varepsilon_{ep\theta}^{(1)}+\frac{1-\mu}{2}a_{33}(\varepsilon_{ep\varphi\theta}^{(1)})^2\Big]dv$$

$$V_{sep}^{(3)}=C\int_V[a_{11}\varepsilon_{p\varphi}^{(1)}\varepsilon_{ep\varphi}^{(1)}+a_{22}\varepsilon_{p\theta}^{(1)}\varepsilon_{ep\theta}^{(1)}+(1-\mu)a_{33}\varepsilon_{ep\varphi\theta}^{(1)}\varepsilon_{p\varphi\theta}^{(1)}$$

$$+\mu a_{12}(\varepsilon_{p\varphi}^{(1)}\varepsilon_{ep\theta}^{(1)}+\varepsilon_{p\theta}^{(1)}\varepsilon_{\theta p\varphi}^{(1)})]dv$$

where, $V_{si}^{(j)}=V_{se}^{(j)}$, $V_{sp}^{(j)}$ expresses the strain energy of prebuckling and buckling state respectively, $j=1,2,12$ express linear, quadratic and higher order item of strain energy, $V_{sep}^{(j)}$ $(j=1,\ 2,\ 3)$ are strain energy consisting of the energy of prebuckling and buckling. $C=E/(1-\mu^2)$, Z is the distance of any point from shell midsurface.

3.2 Strain energy of the meridional stiffeners

The strain energy of the meridional stiffeners can be written as $V_\varphi=\sum_c V_{\varphi c}$, $V_{\varphi c}$ is the strain energy of m-th meridional stiffener and can be resolved using Eq. (3.2):

$$V_{\theta mi}^{(1)}=\frac{E}{2}\int_V a_m[(\varepsilon_{im\varphi}^{(1)})^2+2(Z_m+Z)\varepsilon_{im\varphi}^{(1)}k_{im\varphi}+(Z_m+Z)^2k_{im\varphi}^2]dv$$

$$V_{\theta mi}^{(2)}=\frac{E}{2}\int_V a_m(\varepsilon_{im\varphi}^{(2)})^2dv$$

$$V_{\theta mi}^{(12)}=E\int_V a_m\varepsilon_{im\varphi}^{(2)}[\varepsilon_{im\varphi}^{(1)}+(Z_m+Z)k_{im\varphi}]dv$$

$$V_{\theta mep}^{(1)}=E\int_V a_m[\varepsilon_{emp}(\varepsilon_{pm\varphi}^{(1)}+\varepsilon_{epm\varphi}^{(1)})+(Z_m+Z)(\varepsilon_{em\varphi}k_{pm\varphi}$$

$$+k_{em\varphi}\varepsilon_{pm\varphi}^{(1)}+k_{em\varphi}\varepsilon_{epm\varphi}^{(1)})+(Z_m+Z)^2k_{em\varphi}k_{pm\varphi}]dv$$

$$V_{\theta mep}^{(2)}=\frac{E}{2}\int_V a_m[(\varepsilon_{epm\varphi}^{(1)})^2+2\varepsilon_{em\varphi}\varepsilon_{pm\varphi}^{(2)}+2\varepsilon_{pm\varphi}^{(1)}\varepsilon_{epm\varphi}^{(1)}$$

$$+2(Z_m+Z)(\varepsilon^{(2)}_{\varphi m\varphi}k_{\theta m\varphi}+k_{\varphi m\varphi}\varepsilon^{(1)}_{\theta\varphi m\varphi})]dv$$

$$V^{(3)}_{\theta m\theta\varphi}=E\int_V a_m(\varepsilon^{(2)}_{\varphi m\varphi}\varepsilon^{(1)}_{\theta\varphi m\varphi})dv$$

3.3 Strain energy of the circumferential stiffeners

The strain energy of the circumferential stiffeners can be written as $V_\theta=\sum_m V_{\theta m}$, $V_{\theta m}$ is the strain energy of c-th circumferential stiffener and can be resolved using Eq. (3.2) too. The energy expression of every part of the circumferential stiffener can be obtained through substituting subscript c for m and θ for φ in the energy expressions of the meridional stiffeners.

3.4 Potential energy

Resolving potential energy using Eq. (3.2), and having:

$$U=\sum_{i=c,p}[U^{(1)}_i+U^{(2)}_i+U^{(3)}_i]$$

where

$$U^{(1)}_i=\int_A prw_i dA$$

$$U^{(2)}_i=\frac{1}{2}\int_A p[w_i^2+w_iv_{i\theta}+rw_iu_{i\varphi}+v_i^2-v_iw_{i\theta}-ru_iw_{i\varphi}]dA$$

$$U^{(3)}_i=\frac{1}{6}\int_A p[w_i^2u_{i\varphi}-w_iv_{i\varphi}u_{i\theta}-w_iu_{i\varphi}v_{i\theta}-v_iw_{i\varphi}u_{i\theta}-v_iu_{i\varphi}w_{i\theta}$$
$$+v_i^2u_{i\varphi}+u_iv_{i\varphi}w_{i\theta}-u_iw_iv_{i\varphi}-u_iw_{i\varphi}v_{i\theta}-u_iw_iw_{i\varphi}]dA$$

IV. Buckling Equation and Solution

4.1 Buckling equation

Expanding Eq. (3.1) in (u_e,v_e,w_e), the energy increment of buckling can be obtained:

$$\Delta\Pi=\delta\Pi+\frac{1}{2!}\delta^2\Pi+\frac{1}{3!}\delta^3\Pi+\cdots \tag{4.1}$$

Because the shell is balanced in (u_e,v_e,w_e) and there is $\delta\Pi=0$. The buckling equation of shell can be obtained from Trefftz Theory:

$$\delta(\delta^2\Pi)=\delta(V_p+U_p)=0 \tag{4.2}$$

where,

$$V_p=2V^{(1)}_p=2(V^{(1)}_{sp}+V^{(1)}_{\theta p}+V^{(1)}_{\varphi p})$$

$$U_p=C\int_V\left[(a_{11}\varepsilon_{e\varphi}+\mu a_{12}\varepsilon_{e\theta})\beta_\theta^2+(a_{22}\varepsilon_{e\theta}+\mu a_{12}\varepsilon_{e\varphi})\psi_\theta^2+\frac{1-\mu}{2}\varepsilon_{e\varphi\theta}\beta_\varphi\psi_\varphi\right]dv$$
$$+\sum_m\int_V a_m\left[\varepsilon_{em\varphi}\beta^2_{\varphi m\varphi}\right]dv+\sum_c\int_V a_c[\varepsilon_{ec\theta}\beta^2_{\varphi c\theta}]dv$$

$\varepsilon_{e\varphi}$, $\varepsilon_{e\theta}$, $\varepsilon_{e\varphi\theta}$ and $\varepsilon_{em\varphi}$, $\varepsilon_{ec\theta}$ can be obtained by the prebuckling analysis.

4.2 Displacement function

Because the effect of meridional stiffeners shell shall produce asymmetric deformation before buckling, the shell is a kind of shell of revolution and subjects to uniformly external pressure, the circumferential displacement pattern can be expressed by trigonometric function and the meridional pattern can take a finite combination for some functions. Neglecting meridional and circumferential deformation, the prebuckling displacement pattern can be written as:

$$w_s(\varphi,\theta)=\sum_{i}\phi_{Aj}w_{Aj}+X_w(\theta)\sum_{i}\phi_{Bj}w_{Bj} \tag{4.3}$$

where, ϕ_{Aj} and ϕ_{Bj} are basic functions, w_{Aj} and w_{Bj} are generalized coordinate to be found. $X_w(\theta)$ can take a trigonometric function and let $X_w(\theta)=\cos N\theta$ in this paper, N is the numbers of meridional stiffeners. The basic functions can be taken as arbitrary function which satisfies the boundary condition and deformation feature of the shell.

The displacement of prebuckling sate increases an infinite small amount when buckling and this increment can be expressed as:

$$\left.\begin{aligned} u_p&=[\phi_u]\{u\}=\sum_{k=1}^{P}\phi_{uk}u_k \\ v_p&=[\phi_v]\{v\}=\sum_{j=1}^{S}\phi_{vj}v_j \\ w_p&=[\phi_w]\{w\}=\sum_{m=1}^{Q}\phi_{wm}w_m \end{aligned}\right\} \tag{4.4}$$

according to the feature for shell buckling, the basic function is taken as:

$$\left.\begin{aligned} \phi_{u1}&=\exp[-\beta\varphi^2]\sin^3c_1\varphi\cos n\theta \\ \phi_{v1}&=\exp[-\beta\varphi^2]\sin^3c_2\varphi\sin n\theta \\ \phi_{w1}&=\exp[-\beta\varphi^2][\cos^3c_1\varphi-\sin^3c_2\varphi\cos n\theta] \end{aligned}\right\} \tag{4.5}$$

where,

$c_1=(2m-1)\pi/2\bar{\varphi}$, $c_2=m\pi\sqrt{\varphi}$.

n——the numbers of circumferential waves when buckling,

m——the nubmers of meridional 1/4 waves when buckling,

β——the attenuation coefficient along meridional direction when buckling.

4.3 Solution procedure

Substituting (4.3), (4.4) and (4.5) for the formula (4.2), the buckling problem of stiffened torispherical shell can be changed into following optimization with constraining problem:

$$\left.\begin{aligned} &P_{cr}=\min_{m,n,\beta}P(m,n,\beta,x_s) \\ &\text{s.t.}\quad \delta\Pi|_{x=x_s}=0 \end{aligned}\right\} \tag{4.6}$$

V. Computing Examples

In order to check if the method is correct, some typical problems are calculated. The following are some calculated results.

Example 1 Clamped spherical shell

The clamped spherical shell is subjected to uniform external pressure, with dimensions $R=32\text{cm}$, $t=0.15\text{cm}$, $\bar{\varphi}=60°$, $E=69.629\text{Gpa}$, $\sigma_s=254.973\text{MPa}$, $\mu=0.32$, respectively. The basic data of stress-strain curve of the material are shown in Table 1, the calculation results by some methods are shown in Table 2, the change of buckling pressure with m and n is listed in Table 3

Table 1 Initial data of state-strain curve of the material

$\varepsilon\times10^3$	2.67	3.0	3.5	4.0	4.5	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0
σ(MPa)	196.133	215.746	240.263	257.425	272.625	284.393	301.554	314.107	325.890	333.426	341.271	349.117	356.962

Table 2 Buckling pressure of a clamed spherical shell

method	m	n	P_{cr}(kPa)/β
spherical	—	13	1864.636/—
memberance	2	7	2047.727/5.2261
elastic	2	7	2013.796/5.5262
plastic	3	8	1413.432/17.401

Table 3 The effect of varying the m and n on the buckling pressure (kPa)

n	m	memberance		elastic		plastic	
		P_{cr}	β	P_{cr}	β	P_{cr}	β
7	1	5204.291	0.01628	5116.914	0.01699	1897.293	0.01929
	2	2047.727	5.5489	2013.796	5.5262	1414.511	17.401
	3	2515.013	10.532	2473.335	10.489	1413.629	30.107
	4	3722.310	10.830	3660.626	10.786	1494.239	10.365
8	1	4164.002	0.04178	4093.786	0.04161	1598.092	0.01929
	2	2054.984	5.5226	2020.954	5.2269	1414.903	20.088
	3	2953.959	9.0449	2905.024	9.0461	1413.432	17.401
	4	4428.389	4.2918	4355.035	4.2924	1671.347	4.173
9	1	3428.503	0.01656	3370.840	0.01656	1433.634	0.01929
	2	2168.937	4.8666	2132.946	4.8673	1413.923	17.401
	3	3475.183	7.1849	3417.519	7.1859	1441.774	6.9228
	4	5084.062	1.9896	5004.137	2.4873	1860.027	2.1154

Table 4 Buckling pressure of a clamped torispherical shell

method	P_{cr}(kPa)	n	m	β
spherical	6381.111	4	2	8.346
elastic	4915.693	4	4	13.430
plastic	2015.465	5	4	11.358

Example 2 Clamped torispherical shell

The geometry form of torispherical shell is shown in Fig. 1(a), and some parameters are $r_1=8\text{cm}$, $L=32.1\text{cm}$, $t=0.28\text{cm}$, $R=20\text{cm}$, $H=11.2\text{cm}$, $\varphi_0=30°$, $\mu=0.32$, respectively. The physical property of material is the same as Example 1. The buckling analysis results are shown in Table 4, a series of the results by changing r_1/H is listed in Table 5.

Table 5 The effect of varying the r_1/H on the buckling pressure (kPa)

r_1/H	torispherical			equivalent spherical			spherical
	P_{cr}	n	m	P_{cr}	n	m	
0.0	13040.433	4	2	13040.433	4	2	12096.711
0.25	11235.393	5	3	11776.905	5	2	11009.763
0.50	8271.302	5	3	9548.662	4	2	9215.710
0.714	4915.693	4	4	6381.111	4	2	6490.198
0.75	4257.148	4	4	5664.098	4	2	5824.393
0.85	2657.627	5	4	3280.660	4	2	3591.441
0.90	1949.443	6	4	1916.972	4	2	2239.623
0.95	1513.683	7	6	632.411	4	2	837.558
0.99	1062.619	12	3	26.277	4	2	49.050

On the condition that H and R are definite, the torispherical shell may have different external outline. An extreme form is the radius of hoop shell which equals to zero and the radius of spherical shell obtains minimum. Another extreme form is the radius of spherical which obtains maximum and the radius of hoop maximum (equals the height H of torispherical shell) too. Between the two forms, the shell may have many kinds of radius combinations for spherical and hoop shell. Because the radius of spherical and hoop shell has following relation:

$$L=(R^2+H^2-2Rr_1)/2(H-r_1)$$

When radius r_1 of hoop shell is given the radius L of spherical shell may get. After R and H is definite, the buckling pressure of torispherical shell will decrease rapidly along with increment of radius of hoop shell as shown in Table 5. When r_1/H equals 0, P_{cr} is maximum and r_1/H equals 1, minimum. In the range of 0 and 0.9, the buckling pressure of torispherical shell is lower than that of equivalent spherical shell. When equals 0.9, the buckling pressures of two kinds of are almost equals, and then the pressure of torispherical shell will increase along with the increase of r_1/H and is greater than the equivalent spherical shell. But for this case, the radius of spherical shell is very large and the shell will tend to a flat, the buckling pressure is very lower, it is impossible to use this kind of shell as a head of vessels subjected external pressure in engineering. Following conclusion can be obtained by above analysis, the buckling pressure of torispherical shell is lower than that of equivalent spherical shell in the range of engineering applying. It will give a danger result if using the method of equivalent spherical shell to design torispherical shell.

Example 3 Clamped stiffened torispherical shell

In order to check the method, a buckling experiment of stiffened torispherical shell is completed, the stiffened form of the models is shown in Fig. 1(b), for the model 1, the width of stiffener is 5.5mm, the height of stiffener is 7.3mm, for the model 2, the width and height are 5.5mm and 7.4mm, respectivly other parameters are the same as Example 2. The experimental results and buckling pressure calculated by the method in this paper are listed in Table 6.

Table 6 Buckling pressure of stiffened torispherical Shell

number of model	method	P_{cr}(kPa)	n	m	β	relative error with experiment
1	elastic	9215.799	7	3	8.346	209.1%
	plastic	3166.077	3	4	5.3123	6.2%
	experiment	2981.222	—	—		
2	elastic	9487.395	7	3	8.351	207.8%
	plastic	3258.353	3	4	5.322	5.7%
	experiment	3082.624	—	—		

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