

# AN ITERATION ALGORITHM FOR SOLVING POSTBUCKLING EQUILIBRIUM PATH OF SIMPLY-SUPPORTED RECTANGULAR PLATES UNDER BIAXIAL COMPRESSION\*

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## Abstract

*In this paper, on the basis of von Kármán large deflection equations and its double trigonometric series solution, we present a simple, fast and effective iteration algorithm for solving simply-supported rectangular plate subjected to biaxial compression.*

**Key words** rectangular plates, postbuckling equilibrium path, iteration algorithm

## I. Introduction

The investigation of behaviour of initial buckling and postbuckling equilibrium path for thin plate structures plays a very important role in understanding and making full use of its potential load-carrying capacities behaviour. During the last two decades, the postbuckling behaviour of rectangular plate under uniaxial compression has been investigated in a number of papers, for instance, references [1–5]. In these studies, by taking several terms trigonometric trial functions, Galerkin's method or Energy principle, or Weighted Residuals method is used to derive the nonlinear algebraic equations satisfied by the coefficients of the trial functions, and then the second-order or the fourth-order or the sixth-order asymptotic solution of postbuckling equilibrium path is obtained approximately by applying Thompson's generalized coordinate parameter perturbation method to the nonlinear algebraic equations; or the solution is obtained directly by means of Newton-Raphson iteration method applied to nonlinear algebraic equations which are satisfied by the coefficients of trigonometric series solution. By contrast, however, little work has been done into the analysis of postbuckling behaviour of rectangular plate under biaxial compression. In Ref. [7], Energy principle is applied to the studies of postbuckling problem for the simply-supported rectangular plate subjected to biaxial compression by taking only one term trial function. While in [8], a fourth-order approximate solution of postbuckling equilibrium path is given by means of perturbation method with the nondimensional maximum deflection as perturbation parameter.

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The fourth-order approximate solution, however, does not possess a symmetrical property for the case of the plate subjected to biaxial compression (cf. Section IV). In this paper, a simple and fast iteration algorithm is presented to solve the postbuckling equilibrium path for simply-supported rectangular plate under biaxial compression. Our computation results show that the algorithm of this paper is also very effective, and confirm the symmetrical property mentioned above (cf. Section IV).

## II. Fundamental Equations

Suppose a simply-supported rectangular plate is subjected to biaxial compression. We choose the following Cartesian coordinates as shown in Fig. 1, where  $a$ ,  $b$  is the length and width of the plate, and  $p_x$ ,  $p_y$  denotes the uniform compression along  $x$  and  $y$  direction, respectively. In the following, we denote  $\alpha$ ,  $\beta$  as load ratio and plate aspect ratio resp., i.e.,  $\alpha = p_y/p_x$ ,  $\beta = a/b$ . The large deflection behaviour is described by the following well-known von Kármán equations:

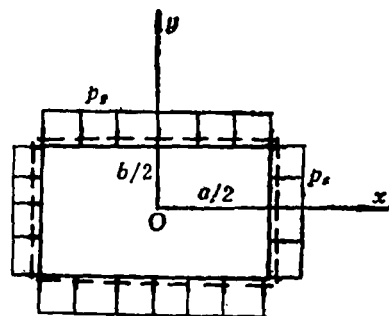


Fig. 1

$$\left. \begin{aligned} D\nabla^4 w &= \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 \varphi}{\partial y^2} - 2 \frac{\partial^2 w}{\partial x \partial y} \frac{\partial^2 \varphi}{\partial x \partial y} + \frac{\partial^2 w}{\partial y^2} \frac{\partial^2 \varphi}{\partial x^2} \\ \nabla^4 \varphi &= Eh \left[ \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right] \end{aligned} \right\} \quad (2.1)$$

where

$$\nabla^4 \equiv \frac{\partial^4}{\partial x^4} + 2 \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}, \quad D \equiv Eh^3/12(1-\nu^2)$$

is the flexural rigidity,  $E$ ,  $\nu$ ,  $h$  is the modulus of elasticity, Poisson's ratio and thickness, of the plate, resp.,  $w$  and  $\varphi$  denote deflection function and stress function. The boundary conditions for the simply-supported rectangular plate subjected to biaxial compression are as follows:

$$\left. \begin{aligned} w = \frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 \varphi}{\partial x \partial y} = 0, \quad \int_{-b/2}^{b/2} \frac{\partial^2 \varphi}{\partial y^2} dy &= -p_x b, \quad \text{at } x = \pm a/2 \\ w = \frac{\partial^2 w}{\partial y^2} = \frac{\partial^2 \varphi}{\partial x \partial y} = 0, \quad \int_{-a/2}^{a/2} \frac{\partial^2 \varphi}{\partial x^2} dx &= -p_y a, \quad \text{at } y = \pm b/2 \end{aligned} \right\} \quad (2.2)$$

Introducing the following nondimensional variables:

$$w = \frac{w}{h}, \quad \bar{x} = \frac{x}{a}, \quad \bar{y} = \frac{y}{b}, \quad \bar{\varphi} = \frac{\varphi}{Eh^3\beta^2}, \quad \bar{p}_x = \frac{b^2 p_x}{Eh^3\beta^2}, \quad \bar{p}_y = \frac{a^2 p_y}{Eh^3\beta^2}$$

Then the nondimensional form of (2.1)–(2.2) may be written as (here and hereafter the sign “~” over each variable has been omitted):

$$\nabla^4 w = \lambda \left( \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 \varphi}{\partial y^2} - 2 \frac{\partial^2 w}{\partial x \partial y} \frac{\partial^2 \varphi}{\partial x \partial y} + \frac{\partial^2 w}{\partial y^2} \frac{\partial^2 \varphi}{\partial x^2} \right)$$

$$\left. \nabla^4 \varphi = \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right\} \quad (2.3)$$

$$\left. \begin{aligned} w = \frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 \varphi}{\partial x \partial y} = 0, \quad \int_{-1/2}^{1/2} \frac{\partial^2 \varphi}{\partial y^2} dy = -p, \quad \text{at } x = \pm \frac{1}{2} \\ w = \frac{\partial^2 w}{\partial y^2} = \frac{\partial^2 \varphi}{\partial x \partial y} = 0, \quad \int_{-1/2}^{1/2} \frac{\partial^2 \varphi}{\partial x^2} dx = -p, \quad \text{at } y = \pm \frac{1}{2} \end{aligned} \right\} \quad (2.4)$$

where

$$\nabla^4 \equiv \frac{\partial^4}{\partial x^4} + 2\beta^2 \frac{\partial^4}{\partial x^2 \partial y^2} + \beta^4 \frac{\partial^4}{\partial y^4}, \quad \lambda \equiv 12(1-\nu^2)\beta^4$$

Thus, (2.3)–(2.4) can be used to study the postbuckling behaviour of simply-supported rectangular plate subjected to biaxial compression.

### III. Double Trigonometric Series Solution and Iteration Algorithm

By means of the method of double trigonometric series, the solution to (2.3)–(2.4) may be assumed as follows:

$$\left. \begin{aligned} w = w(x, y) &= \sum_{m=1,3}^{\infty} \sum_{n=1,3}^{\infty} w_{mn} \cos m\pi x \cos n\pi y \\ \varphi = \varphi(x, y) &= \frac{1}{2} (\bar{p}_s y^2 + \bar{p}_r x^2) - \frac{1}{8} \sum_{r=0,2}^{\infty} \sum_{s=0,2}^{\infty} \varphi_{rs} \cos r\pi x \cos s\pi y \end{aligned} \right\} \quad (3.1)$$

here  $w_{mn}$ ,  $\bar{p}_s$ ,  $\bar{p}_r$  and  $\varphi_{rs}$  are undetermined constants. Substituting (3.1) into (2.3b), by direct computation, we derive that  $\varphi_{rs}$  are functions of  $w_{mn}$  as follows:

$$\left. \begin{aligned} \varphi_{0,0} &= 0 \\ \varphi_{r,0} &= \frac{1}{r^2} \sum_{q=1,3}^{\infty} q^2 \left[ 2 \sum_{p=1,3}^{\infty} w_{pq} w_{r+p,q} + \sum_{p=1,3}^{r-1} w_{pq} w_{r-p,q} \right] \quad (r=2,4,\dots) \\ \varphi_{0,s} &= \frac{1}{\beta^4 s^2} \sum_{q=1,3}^{\infty} p^2 \left[ 2 \sum_{q=1,3}^{\infty} w_{pq} w_{p,s+q} + \sum_{q=1,3}^{s-1} w_{pq} w_{p,s-q} \right] \quad (s=2,4,\dots) \\ \varphi_{r,s} &= \frac{1}{(r^2 + \beta^2 s^2)^2} \left\{ \sum_{p=1,3}^{r-1} \left[ \sum_{q=1,3}^{s-1} (rq - sp)^2 w_{pq} w_{r-p,s-q} \right. \right. \\ &\quad \left. \left. + 2 \sum_{q=1,3}^{\infty} (rq + sp)^2 w_{pq} w_{r-p,s+q} \right] + 2 \sum_{p=1,3}^{\infty} \left[ \sum_{q=1,3}^{s-1} (rq + sp)^2 w_{pq} w_{r+p,s-q} \right. \right. \\ &\quad \left. \left. + 2 \sum_{q=1,3}^{\infty} (rq - sp)^2 w_{pq} w_{r+p,s+q} \right] \right\} \quad (r=2,4,\dots, s=2,4,\dots) \end{aligned} \right\} \quad (3.2)$$

Inserting (3.1) into (2.3a), yields the following nonlinear algebraic equations governing  $w_{mn}$ :

$$32D_{mn}w_{mn} + R_{mn} = 0 \quad (m=1,3,\dots; n=1,3,\dots) \quad (3.3)$$

where

$$D_{mn} = \frac{(m^2 + \beta^2 n^2)^2}{\lambda} + \frac{(m^2 \bar{p}_s + n^2 \bar{p}_r)}{\pi^2} \quad \left( \begin{matrix} m=1,3,\dots \\ n=1,3,\dots \end{matrix} \right) \quad (3.4)$$

$$\begin{aligned} R_{mn} = & \sum_{p=1,3}^m \left[ \sum_{q=1,3}^n (np - mq)^2 w_{pq} \varphi_{m-p, n-q} + \sum_{q=1,3}^{\infty} (np + mq)^2 w_{pq} \varphi_{m-p, n+q} \right] \\ & + \sum_{p=1,3}^{\infty} \left[ \sum_{q=1,3}^n (np + mq)^2 w_{pq} \varphi_{m+p, n-q} + \sum_{q=1,3}^{\infty} (np - mq)^2 w_{pq} \varphi_{m+p, n+q} \right] \\ & + \sum_{r=0,2}^{\infty} \left[ \sum_{q=1,3}^{\infty} (mn + mq + nr)^2 w_{m+r, q} \varphi_{r, n+q} + \sum_{q=1,3}^n (mn + nr - mq)^2 w_{m+r, q} \varphi_{r, n-q} \right] \\ & + \sum_{s=0,2}^{\infty} \left[ \sum_{p=1,3}^{\infty} (mn + ms + np)^2 w_{p, n+s} \varphi_{m+p, s} + \sum_{p=1,3}^m (mn + ms - np)^2 w_{p, n+s} \varphi_{m-p, s} \right] \\ & + \sum_{r=0,2}^{\infty} \sum_{s=0,2}^{\infty} (ms - nr)^2 w_{m+r, n+s} \varphi_{rs} \quad (m=1,3,\dots; n=1,3,\dots) \end{aligned} \quad (3.5)$$

By substitution of (3.1) into (2.4), it is ready to verify that the first three conditions of (2.4a) and (2.4b) are satisfied, and to yield from the last conditions (2.4a) and (2.4b) that:

$$\bar{p}_s = -p_s, \quad \bar{p}_r = -p_r \quad (3.6)$$

The exact double trigonometric series solution to (2.3)–(2.4) is therefore obtained, provided that  $w_{mn}$  are solved from (3.3). Since (3.3) are a set of infinite dimensional-equations, it is therefore required to confine us to solve only a set of limited number equations in actual computation. The solution is then checked whether it already approaches the exact solution of (3.3) by increasing the number of equations considered. Without loss of generality, we assume the largest subscripts  $m$  and  $n$  are  $2M-1$  and  $2N-1$ , respectively. For the sake of programming convenience, we denote

$$\left. \begin{aligned} w(i, j) &\equiv w_{2i-1, 2j-1}, \quad \varphi(i, j) \equiv \varphi_{2(i-1), 2(j-1)} \\ R(i, j) &\equiv R_{2i-1, 2j-1}, \quad D(i, j) \equiv D_{2i-1, 2j-1} \end{aligned} \right\} \quad (3.7)$$

(3.2)–(3.5) then may be rewritten as:

$$\varphi(1, 1) = 0$$

$$\begin{aligned} \varphi(i, 1) = & B(i, 1) \sum_{l=1}^N (2l-1)^2 \left[ \sum_{k=1}^{i-1} w(k, l) w(i-k, l) \right. \\ & \left. + 2 \sum_{k=1}^{M-i+1} w(k, l) w(i+k-1, l) \right] \quad (i=2, 3, \dots, M) \end{aligned}$$

$$\varphi(i, 1) = B(i, 1) \sum_{l=1}^N (2l-1)^2 \sum_{k=i-M}^M w(k, l) w(i-k, l)$$

$$(i = M+1, M+2, \dots, 2M)$$

$$\varphi(1, j) = B(1, j) \sum_{k=1}^M (2k-1)^2 \left[ \sum_{l=1}^{j-1} w(k, l) w(k, j-l) \right. \\ \left. + 2 \sum_{l=1}^{N-j+1} w(k, l) w(k, j+l-1) \right] \quad (j=2, 3, \dots, N)$$

$$\varphi(1, j) = B(1, j) \sum_{k=1}^M (2k-1)^2 \sum_{l=j-N}^N w(k, l) w(k, j-l)$$

$$(j = N+1, N+2, \dots, 2N)$$

$$\varphi(i, j) = B(i, j) \left\{ \sum_{k=1}^{i-1} \left[ \sum_{l=1}^{j-1} [(i-1)(2l-1) - (j-1)(2k-1)]^2 w(k, l) w(i-k, j-l) \right. \right. \\ \left. + 2 \sum_{l=1}^{N-j+1} [(i-1)(2l-1) + (j-1)(2k-1)]^2 w(k, l) w(i-k, j+l-1) \right] \\ \left. + 2 \sum_{k=1}^{M-i+1} \left[ \sum_{l=1}^{j-1} [(i-1)(2l-1) + (j-1)(2k-1)]^2 w(k, l) w(i+k-1, j-l) \right. \right. \\ \left. + 2 \sum_{l=1}^{N-j+1} [(i-1)(2l-1) - (j-1)(2k-1)]^2 w(k, l) w(i+k-1, j+l-1) \right] \Big\}$$

$$(i=2, 3, \dots, M, j=2, 3, \dots, N)$$

$$\varphi(i, j) = B(i, j) \left\{ \sum_{k=i-N}^N \left[ \sum_{l=1}^{i-1} [(i-1)(2l-1) - (j-1)(2k-1)]^2 w(k, l) w(i-k, j-l) \right. \right. \\ \left. + 2 \sum_{k=1}^{M-i+1} [(i-1)(2l-1) + (j-1)(2k-1)]^2 w(k, l) w(i+k-1, j-l) \right] \Big\}$$

$$(i=2, 3, \dots, M, j=N+1, N+2, \dots, 2N)$$

$$\varphi(i, j) = B(i, j) \left\{ \sum_{k=i-M}^M \left[ \sum_{l=1}^{j-1} [(i-1)(2l-1) - (j-1)(2k-1)]^2 w(k, l) w(i-k, j-l) \right. \right. \\ \left. + 2 \sum_{l=1}^{N-j+1} [(i-1)(2l-1) + (j-1)(2k-1)]^2 w(k, l) w(i-k, j+l-1) \right] \Big\}$$

$$(i=M+1, M+2, \dots, 2M, j=2, 3, \dots, N)$$

$$\varphi(i, j) = B(i, j) \sum_{k=i-M}^M \sum_{l=j-N}^N [(i-1)(2l-1) - (j-1)(2k-1)]^2 w(k, l) w(i-k, j-l)$$

$$(i=M+1, M+2, \dots, 2M, j=N+1, N+2, \dots, 2N)$$

$$32D(i, j)w(i, j) + R(i, j) = 0 \quad (i=1, 2, \dots, M; j=1, 2, \dots, N) \quad (3.9)$$

$$D(i, j) = \frac{[(2i-1)^2 + \beta^2(2j-1)^2]^2}{\lambda} - \frac{(2i-1)^4 p_x + (2j-1)^4 p_y}{\pi^4} \quad (3.10)$$

$$\begin{aligned} R(i, j) = & \sum_{k=1}^i \left\{ \sum_{l=1}^j [(2j-1)(2k-1) - (2i-1)(2l-1)]^2 w(k, l) \varphi(i-k+1, j-l+1) \right. \\ & + \sum_{l=1}^N [(2j-1)(2k-1) + (2i-1)(2l-1)]^2 w(k, l) \varphi(i-k+1, j+l) \\ & + \sum_{l=1}^{N-j+1} [(2i-1)(2j+2l-3) - (2j-1)(2k-1)]^2 w(k, j+l-1) \varphi(i-k+1, l) \Big\} \\ & + \sum_{k=1}^M \left\{ \sum_{l=1}^j [(2j-1)(2k-1) + (2i-1)(2l-1)]^2 w(k, l) \varphi(i+k, j-l+1) \right. \\ & + \sum_{l=1}^N [(2j-1)(2k-1) - (2i-1)(2l-1)]^2 w(k, l) \varphi(i+k, j+l) \\ & + \sum_{l=1}^{N-j+1} [(2i-1)(2j+2l-3) + (2j-1)(2k-1)]^2 w(k, j+l-1) \varphi(i+k, l) \Big\} \\ & + \sum_{k=1}^{M-i+1} \left\{ \sum_{l=1}^j [(2j-1)(2i+2k-3) \right. \\ & \quad \left. - (2i-1)(2l-1)]^2 w(i+k-1, l) \varphi(k, j-l+1) \right. \\ & + \sum_{l=1}^N [(2i+2k-3)(2j-1) + (2i-1)(2l-1)]^2 w(i+k-1, l) \varphi(k, j+l) \\ & + 4 \sum_{l=1}^{N-j+1} [(2i-1)(l-1) - (2j-1)(k-1)]^2 \\ & \quad \left. \cdot w(i+k-1, j+l-1) \varphi(k, l) \right\} \end{aligned} \quad (3.11)$$

here we have used the following notations:

$$\left. \begin{aligned} B(1, j) &\equiv 1/4\beta^4(j-1)^2 & (j=2, 3, \dots, 2N) \\ B(i, 1) &\equiv 1/4(i-1)^2 & (i=2, 3, \dots, 2M) \\ B(i, j) &\equiv 1/4[(i-1)^2 + \beta^2(j-1)^2]^2 & (i=2, 3, \dots, 2M; j=2, 3, \dots, 2N) \end{aligned} \right\} \quad (3.12)$$

In the analysis of postbuckling behaviour of plate and shell structures, one of most interesting problems is to find out the relation between applied loads and the maximum deflection, that is the so-called postbuckling equilibrium path. It is, however, impossible to obtain a closed-form analytical solution. Seide proposed an iteration procedure in [9-10] for solving large deflection bending problem of rectangular plate and membranes. Based on Seide's idea, we apply the following iteration algorithm for directly solving the postbuckling

equilibrium path. Before we introduce the iteration algorithm, first, we note that the maximum deflection occurs at the center of the plate because of symmetrical property. From (3.1), we therefore have

$$w_o = w(0, 0) = \sum_{i=1}^M \sum_{j=1}^N w(i, j) \quad (3.13)$$

On the other hand, we remark that there is the following relation between nondimensional compression  $p_o$  and  $p_r$ :

$$p_r = \alpha \beta^2 p_o \quad (3.14)$$

Using (3.13)–(3.14), we denote  $w_o(I)$  ( $I=1, 2, \dots, II$ ) as the different value of the center deflection, and  $p_o(I)$  and  $w^{(I)}(i, j)$  are the corresponding value when the value of center deflection is  $w_o(I)$  (usually,  $w_o(II) \leq 3.0$ ). Thus, the iteration algorithm of this paper may be expressed as follows:

$$\left\{ \begin{array}{l} w^{(0,1)}(i, j) = 0, \quad w^{(0,1)}(1, 1) = w_o(1) \\ p_o^{(0)}(1) = -\frac{\pi^2(1+\beta^2)^2}{\lambda(1+\alpha\beta^2)} \left[ 1 + \frac{\lambda}{32(1+\beta^2)^2} \frac{R^{(0,1)}(1, 1)}{w^{(0,1)}(1, 1)} \right] \\ w^{(1,1)}(i, j) = -\frac{R^{(0,1)}(i, j)}{32D^{(0,1)}(i, j)} \quad (i+j \geq 2) \\ w^{(1,1)}(1, 1) = w_o(1) - \sum_{i=1}^M \sum_{j=1}^N w^{(1,1)}(i, j) \\ p_o^{(1)}(1) = -\frac{\pi^2(1+\beta^2)^2}{\lambda(1+\alpha\beta^2)} \left[ 1 + \frac{\lambda}{32(1+\beta^2)^2} \frac{R^{(1,1)}(1, 1)}{w^{(1,1)}(1, 1)} \right] \\ \\ w^{(0,I)}(i, j) = w^{(I-1)}(i, j) + [w^{(I-1)}(i, j) - w^{(I-2)}(i, j)] \\ w^{(0,I)}(1, 1) = w_o(I) - \sum_{i=1}^M \sum_{j=1}^N w^{(0,I)}(i, j) \\ p_o^{(0)}(I) = -\frac{\pi^2(1+\beta^2)^2}{\lambda(1+\alpha\beta^2)} \left[ 1 + \frac{\lambda}{32(1+\beta^2)^2} \frac{R^{(0,I)}(1, 1)}{w^{(0,I)}(1, 1)} \right] \\ w^{(1,I)}(i, j) = -\frac{R^{(0,I)}(i, j)}{32D^{(0,I)}(i, j)} \quad (i+j \geq 2) \\ w^{(1,I)}(1, 1) = w_o(I) - \sum_{i=1}^M \sum_{j=1}^N w^{(1,I)}(i, j) \\ p_o^{(1)}(I) = -\frac{\pi^2(1+\beta^2)^2}{\lambda(1+\alpha\beta^2)} \left[ 1 + \frac{\lambda}{32(1+\beta^2)^2} \frac{R^{(1,I)}(1, 1)}{w^{(1,I)}(1, 1)} \right] \end{array} \right.$$

Here we remark that  $p_o(I)$  and  $w^{(I)}(i, j)$  defined above are identical to the convergence values of  $p_o^{(b)}(I)$  and  $w^{(b,I)}$ , respectively.

#### IV. Analysis and Comparison of Computation Results

From the iteration algorithm presented in the last section, compared with Newton-Raphson's iteration method, we see our algorithm is very simple and fast because it is not involved in computing Jacobian matrix and solving linear algebraic equations. Based on the above iteration algorithm, the detailed computations are carried out at VAX—11/750 computer. The computation results demonstrate that the algorithm is also fast and effective (Convergence criterion governing parameter is taken  $10^{-6}$  and Poisson's ratio equals to 0.3, and the values of  $M$  and  $N$  are considered to be equal). The variation of iteration solutions with the increasing number of equations for  $\alpha=1$  and  $\beta=2$  is presented in Table 1 in which the sign"—" denotes the iteration solution has already converged. It is seen from Table 1 that the results obtained approximately by taking only several terms trigonometric functions as an assumed form of deflection functions are not accurate enough, especially when the center deflection is relatively large. It should be noted that, for the cases of the length, the width and load ratio taken as  $a, b, \alpha$  and  $b, a, 1/\alpha$  respectively, from symmetrical analysis, or from the view of rotating the coordinate axes 90 degree, these two cases are in fact identical, and the postbuckling equilibrium paths can be considered as the same besides a constant factor. Using the nondimensional variables of this paper, if we denote  $p_z^*$  and  $p_z^{**}$  are the compression corresponding to center deflection  $w_0$  with the parameter as  $(\alpha, \beta)$  and  $(1/\alpha, 1/\beta)$ . Then it is easy to verify there exists the relation  $p_z^{**} = \alpha\beta^6 p_z^*$ . But the result presented in [8] (Eq. (25)) does not have such a symmetrical property. The computation results in Table 2 confirm the

**Table 1** Variation of iteration solutions of compression  $p_z/p_c$  with increasing number of equations and the number of iteration cycles when  $M=9$  ( $p_c=0.28244$ ,  $\alpha=1$ ,  $\beta=2$ , and iteration step is 0.1)

$w_c$ \ $M=N$	$M=1$	$M=2$	$M=3$	$M=5$	$M=6$	$M=8$	$M=9$	Number of iteration cycles
0.1	1.00464	—	—	—	—	—	—	2
0.5	1.11602	1.11377	1.11369	—	—	—	—	4
1.0	1.46410	1.43999	1.43475	1.43489	—	—	—	6
1.5	2.04422	1.99079	1.94520	1.94698	1.94699	—	—	5
2.0	2.85640	2.79687	2.63863	2.64521	2.64524	—	—	6
2.5	3.90062	3.84297	3.49346	3.50520	3.50506	3.50509	—	7
3.0	5.17690	5.08442	4.48421	4.48933	4.48781	4.48789	—	10

**Table 2** Iteration solution of compression  $p_z/p_c$  when the parameters are  $\alpha, \beta$  and  $1/\alpha, 1/\beta$  ( $M=N=5$ )

$w_c$	$\alpha=2.0$ $\beta=0.5$	$\alpha=0.5$ $\beta=2.0$	$\alpha=1.0$ $\beta=0.5$	$\alpha=1.0$ $\beta=2.0$	$\alpha=2.0$ $\beta=1.0$	$\alpha=0.5$ $\beta=1.0$
0.1	1.00464	—	1.00464	—	1.00341	—
0.5	1.11286	—	1.11369	—	1.08550	—
1.0	1.42929	—	1.43489	—	1.34420	—
1.5	1.94817	—	1.94698	—	1.78265	—
2.0	2.65189	—	2.64521	—	2.41002	—
2.5	3.49426	—	3.50520	—	3.23208	—
3.0	4.43359	—	4.48933	—	4.23609	—
$p_c$	15.0635	0.470734	18.0762	0.28244	1.20508	2.41016



above property obtained from qualitative analysis. Figs. 2–4 show the computation results of postbuckling equilibrium paths with different load ratio or different aspect ratio. It can be seen that effects of aspect ratio is much sensible than that of load aspect, to the postbuckling equilibrium paths. As a special case of this paper, setting  $\alpha=0$ , the simply-supported rectangular plate subjected to biaxial compression reduces to the plate under uniaxial compressions with the unloaded edges unrestrained. The comparison of results of postbuckling equilibrium paths for a square plate, of present paper and of Ref. [3] are given in Fig. 5. It can be seen that those results are in very good agreement.

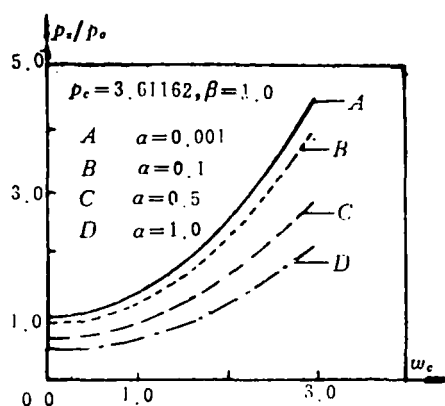


Fig. 2 Compression-center deflection curves for different cases

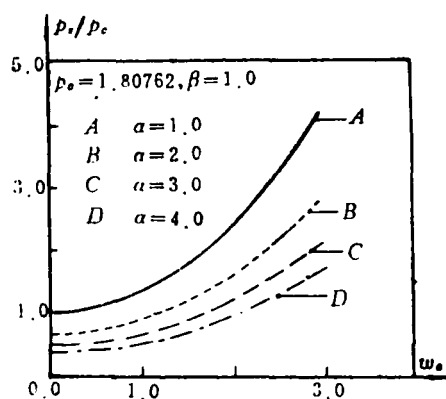


Fig. 3 Compression-center deflection curves for different cases

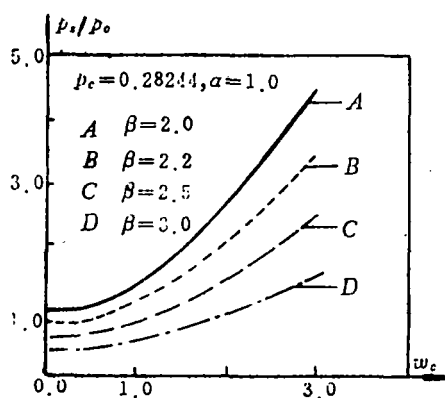


Fig. 4 Compression-center deflection curves for different cases

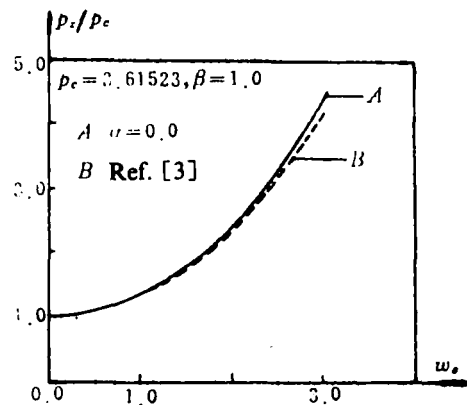


Fig. 5 Comparison of results of Ref. [3] and of the present

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