

THE EFFECT OF TRANSVERSE SHEAR DEFORMATION ON STRESS CONCENTRATION FACTORS FOR SHALLOW SHELLS WITH A SMALL CIRCULAR HOLE*

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Abstract

Simplified equations are derived for the analysis of stress concentration for shear-deformable shallow shells with a small hole. General solutions of the equations are obtained, in terms of series, for shallow spherical shells and shallow circular cylindrical shells with a small circular hole. Approximate explicit solutions and numerical results are obtained for the stress concentration factors of shallow circular cylindrical shells with a small hole on which uniform pressure is acting.

Key words shells, transverse shear deformation, stress concentration

I. Introduction

A system of equations of the linear theory of shear-deformable shallow two-dimensionally isotropic shells was presented by Reissner and Wan^[1]. With the equations, Reissner and Wan^[2] studied the problems of transverse twisting and tangential shearing of shallow spherical shells with a small circular hole, calculated the stress concentration factors for both problems and found that there is great effect of transverse shear deformation on stress concentration.

With Reissner and Wan's equations, it is difficult to study the problems of stress concentration of shallow shells with a small hole. This paper derives a system of simplified equations for the analysis of stress concentration. In the form, the simplified equations are similar to the equations of the theory of thin shallow shells, hence the same methods can be used to solve the equations as are used in the theory of thin shallow shells.

Given a shallow cylindrical shell on which uniform pressure is acting, with midsurface radius \bar{R} , bending stiffness factor D , membrane flexibility factor B , transverse shear flexibility factor A , and a circular hole of radius a , it is found that the stress concentration factors K are functions of just three dimensionless parameters, which is similar to the case of spherical shells (Reissner and was^[2]). The parameters are $\mu = a/(\sqrt{DB}\sqrt{R})$, $\lambda = \sqrt{2/(1-\nu)}a/\sqrt{DA}$ and Poisson ratio ν . For a transversely homogeneous shell of wall thickness h , with a Young's modulus E and a modulus of rigidity G for the transverse shearing stress, we have the expressions of D , B and A

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$$D = \frac{Eh^3}{12(1-\nu^2)}, \quad B = \frac{1}{Eh}, \quad A = \frac{6}{5Gh}$$

and accordingly

$$\mu = \sqrt{\frac{12(1-\nu^2)}{R}} \frac{a}{\sqrt{Rh}}, \quad \lambda = \sqrt{\frac{20(1+\nu)}{E/G}} \frac{a}{h}$$

The limiting case $\lambda = \infty$ corresponds to configurations for which transverse shear deformation effects are absent, so the present results of $K(\lambda, \mu, \nu)$ comprise the results in Xu, Pei and Zhu^[3].

II. Equations for the Analysis of Stress Concentration of Uniform Isotropic Shear-Deformable Shallow Shells with a Small Hole

In the theory of shear-deformable uniform isotropic shallow shells, the expressions of stress resultants and stress couples are

$$N_{11} = K_{,22}, \quad N_{22} = K_{,11}, \quad N_{12} = N_{21} = -K_{,12} \quad (2.1a, b, c)$$

$$Q_1 = -D(\nabla^2 V)_{,1} + \chi_{,2}, \quad Q_2 = -D(\nabla^2 V)_{,2} - \chi_{,1} \quad (2.2a, b)$$

$$\left. \begin{aligned} M_{11} &= -D(V_{,11} + \nu V_{,22}) + (1-\nu)AD\chi_{,12} \\ M_{22} &= -D(V_{,22} + \nu V_{,11}) - (1-\nu)AD\chi_{,12} \\ M_{12} &= -(1-\nu)DV_{,12} + (1-\nu)AD(\chi_{,22} - \chi_{,11})/2 \end{aligned} \right\} \quad (2.3a, b, c)$$

where

$$V = W + AD[\nabla^2 W + A\mathcal{L}K] \quad (2.4)$$

$$\mathcal{L} = Z_{,22}(\quad)_{,11} - 2Z_{,12}(\quad)_{,12} + Z_{,11}(\quad)_{,22}$$

Z represents the middle surface of the shell, and K , W and χ are solutions of the following equations

$$D\nabla^2 \nabla^2 W - \mathcal{L}K + AD\nabla^2 \mathcal{L}K = 0 \quad (2.5)$$

$$B\nabla^2 \nabla^2 K + \mathcal{L}W = 0, \quad (1-\nu)AD\nabla^2 \chi/2 - \chi = 0 \quad (2.6a, b)$$

If the order of magnitude of midsurface radius is R , wall thickness h , and size of the hole a , and our purpose is to study the stress in a small neighborhood of the hole, the orders of magnitude of the first term and third term in Eq. (2.5) are DW/a^4 , $ADK/(Ra^4)$, respectively. With Eq. (2.6a), we have the order-of-magnitude relation $BK/a^4 \sim W/Ra^2$, and then the ratio $(5/12(1+\nu)) \cdot (R/a)^2$ of order-of-magnitude of the first term and third terms in Eq. (2.5). Generally speaking, in practical engineering we have $(R/a) \gg 1$ and $(5/(12(1+\nu))) \cdot (R/a)^2 \gg 1$. With this the third term in Eq. (2.5) can be neglected and Eq. (2.5) is simplified into

$$D\nabla^2 \nabla^2 W - \mathcal{L}K = 0 \quad (2.7)$$

The state of stress of a shell with a small hole consists of two parts. One part is the state of stress which holds in the absence of the hole. The other is the variation of stress produced by the presence of the hole, which approaches rapidly to zero as the distance increases between the point studied and the hole, that is to say, it exists only in a small neighborhood of the hole. Eqs. (2.6a,b) and (2.7) instead of Eqs. (2.6a,b) and (2.5) can be used to study the variation of stress only if a/R is small because Eq. (2.7) is valid in the neighborhood of the hole. In the form Eqs. (2.6a) and (2.7) are equivalent to the equations of thin shallow shells, hence, to solve Eqs. (2.6a) and (2.7), we can use the methods used in the theory of thin shallow shells.

III. Solutions of Shallow Spherical Shells with a Small Circular Hole

Given a shallow spherical shell with midsurface radius R and circular hole of radius a , with the method used in Reissner^[4], W and K satisfying Eqs. (2.6a) and (2.7) can be expressed in terms of two harmonic functions ψ and ϕ and in terms of a function ω , which is the solution of a fourth-order equation as follows.

$$W = \omega + \psi, \quad K = \phi - RD\nabla^2\omega \quad (3.1a, b)$$

$$R^2BD\nabla^2\nabla^2\omega + \omega = 0 \quad (3.2)$$

From now on the problems will be considered in a polar coordinate system (r, θ) . We introduce a nondimensional radial coordinate ρ and two nondimensional parameters λ and μ through the expressions

$$\rho = \frac{r}{a}, \quad \lambda = \sqrt{10} \frac{a}{h}, \quad \mu = \sqrt{12(1-\nu^2)} \frac{a}{\sqrt{Rh}}$$

If our purpose is only to study the variation of stress produced by the presence of the hole and the problem is symmetric about line $\theta = 0$, ω , ϕ , ψ and χ can be expressed as follows.

$$\psi = \sum_{n=1}^{\infty} B_n \rho^{-n} \cos n\theta \quad (3.3a)$$

$$\phi = D_{10}\theta + D_{20} \ln \rho - \frac{a^2 B_{21} \rho}{RB(1+\nu)} [\theta \sin \theta - \ln \rho \cos \theta] + D_{11} \rho \cos \theta + \sum_{n=1}^{\infty} D_{2n} \rho^{-n} \cos n\theta \quad (3.3b)$$

$$\chi = \sum_{n=1}^{\infty} A_n K_n(\lambda \rho) \sin n\theta \quad (3.3c)$$

$$\omega = \sum_{n=0}^{\infty} [C_{1n} H_n^{(1)}(\sqrt{-i} \mu \rho) + C_{2n} K_n(\sqrt{-i} \mu \rho)] \cos n\theta \quad (3.3d)$$

In these expressions B_n , D_{10} , D_{11} , D_{2n} , A_n , C_{1n} and C_{2n} are constants of integration. We can get the solution of the problems symmetrical about $\theta = \pi/2$ by exchanging the positions of cos and sin in Eqs. (3.3a,b,c,d).

An application of the above solution to the case of transverse twisting of shallow spherical shells with a small circular hole gives the stress concentration factors of this problem.

$$K_b = \frac{(1+\nu) \left(1 - \frac{4}{(1-\nu)^2} \left(\frac{\mu}{\lambda}\right)^4\right) - \frac{2(1+\nu)\mu^2}{(1-\nu)\lambda^2} f_2}{\left[1 - \frac{4}{(1-\nu)^2} \left(\frac{\mu}{\lambda}\right)^4\right] \frac{f}{2} - \frac{\mu^2 f}{(1-\nu)\lambda^2} f_2 + \left[(1-\nu) \left(1 - \frac{4}{(1-\nu)^2} \left(\frac{\mu}{\lambda}\right)^4\right) + \frac{(f-1)\mu^4}{(1-\nu)\lambda^2}\right] f_1} \quad (3.4a)$$

$$K_m = \frac{f_2}{\left[1 - \frac{4}{(1-\nu)^2} \left(\frac{\mu}{\lambda}\right)^4\right] \frac{f}{2} - \frac{\mu^2 f}{(1-\nu)\lambda^2} f_2 + \left[(1-\nu) \left(1 - \frac{4}{(1-\nu)^2} \left(\frac{\mu}{\lambda}\right)^4\right) + \frac{(f-1)\mu^4}{(1-\nu)\lambda^2}\right] f_1} \cdot \sqrt{\frac{1-\nu^2}{3}} \quad (3.4b)$$

where

$$f = 2 - \frac{2}{\lambda} \frac{K_1(\lambda)}{K_2(\lambda)}$$

$$g_1 = \text{kei}'(\mu) \text{ker}_2(\mu) - \text{ker}'(\mu) \text{kei}_2(\mu), \quad g_2 = -[\text{kei}'(\mu) \text{kei}(\mu) + \text{ker}'(\mu) \text{ker}(\mu)]$$

$$f_1 = \frac{(\text{ker}'(\mu))^2 + (\text{kei}'(\mu))^2}{-2\mu g_1}, \quad f_2 = \frac{g_2}{g_1}$$

Setting $\lambda \rightarrow \infty$, we have from Eqs. (3.4a,b)

$$K_b = \frac{1+\nu}{1+(1-\nu)f_1}, \quad K_m = \sqrt{\frac{1-\nu^2}{3}} \frac{f_2}{1+(1-\nu)f_1} \quad (3.5a, b)$$

The results in Eqs. (3.5a,b) are consistent with the results in [5]. In practical engineering, the inequalities $a/R < 2/5$ and $h/R < 1/3$ always hold, and in this range the results calculated by Eqs. (3.4a,b) are consistent with the results in [2] (see Fig. 1). This, in some way, shows that the simplified equations are exact enough for problems in practical engineering.

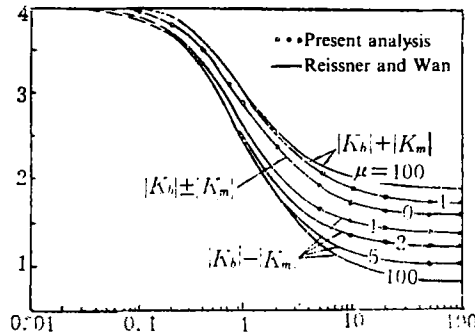


Fig. 1 Upper and lower values of stress concentration factors for the effect of a circular hole, in a shallow spherical shell for the problem of transverse twisting. The curves shown are for $\nu = 0.3$

IV. Solutions of Circular Cylindrical Shells with a Small Circular Hole

We introduce a complex function

$$\sigma = W - iK/\sqrt{DEh} \quad (4.1)$$

With Eq. (4.1) a combination can be made of Eq. (2.6a) with (2.7) and this gives

$$\nabla^2 \nabla^2 \sigma - (i/\sqrt{DB}) \mathcal{L} \sigma = 0 \quad (4.2)$$

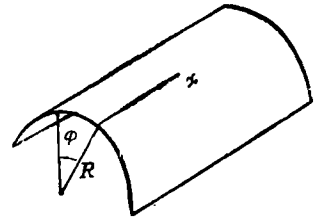


Fig. 2

From now on we will use coordinate (ρ, θ) introduced through the relations

$$x/R = \rho \cos \theta, \quad \varphi = \rho \sin \theta \quad (4.3a, b)$$

where coordinates x and φ are shown in Fig. 2

For problems symmetrical about the line $\theta = 0$, the approximate solution of Eq. (4.2) was presented in Ref. [3].

$$\begin{aligned} \text{Im}\sigma = & \frac{2A_{00}}{\pi} \left(\ln \frac{\rho}{\rho_0} + \ln \frac{\xi \rho_0 \gamma}{\sqrt{2}} \right) - \frac{A_{10}}{\pi} (1 + \cos 2\theta) - \frac{2B_{21}}{\pi} \frac{\cos 2\theta}{\rho^2} \\ & + \xi^2 \left[-\frac{A_{00}\rho^2}{4} (2 + \cos 2\theta) + \frac{A_{10}\rho^2}{4} (1 + \cos 2\theta) + \frac{2A_{01}}{\pi} \left(\ln \frac{\rho}{\rho_0} + \ln \frac{\xi \rho_0 \gamma}{\sqrt{2}} \right) \right. \\ & \left. + \frac{B_{01}}{2} - \frac{2B_{22}}{\pi} \frac{\cos 2\theta}{\rho^2} - \frac{A_{11}}{\pi} (1 + \cos 2\theta) \right] \end{aligned} \quad (4.4a)$$

$$\begin{aligned} \text{Re}\sigma = & \frac{\xi^2}{\pi} \left\{ \frac{\pi A_{01}}{2} - 2B_{01} \ln \frac{\xi \rho_0 \gamma}{\sqrt{2}} + B_{11} - \frac{B_{21}}{2} + A_{00}\rho^2 \left(2 \ln \frac{\rho}{\rho_0} + 2 \ln \frac{\xi \rho_0 \gamma}{\sqrt{2}} - 1 \right) \right. \\ & - 2B_{01} \ln \frac{\rho}{\rho_0} + A_{10}\rho^2 \left(\frac{1}{4} - \ln \frac{\rho}{\rho_0} - \ln \frac{\xi \rho_0 \gamma^2}{\sqrt{2}} \right) + \left[A_{00}\rho^2 \left(\ln \frac{\rho}{\rho_0} + \ln \frac{\xi \rho_0 \gamma}{\sqrt{2}} \right) \right. \\ & \left. + A_{10}\rho^2 \left(\frac{1}{6} - \ln \frac{\rho}{\rho_0} - \ln \frac{\xi \rho_0 \gamma}{\sqrt{2}} \right) + B_{11} - \frac{2A_{22}}{\rho^2} - \frac{4A_{12}}{\rho^2} \right] \cos 2\theta \\ & \left. - \left(\frac{\rho^2}{12} A_{10} + \frac{B_{21}}{2} + \frac{4A_{32}}{\rho^2} + \frac{24B_{43}}{\rho^4} \right) \cos 4\theta \right\} \end{aligned} \quad (4.4b)$$

In these expressions $\rho_0 = a/R$, $\xi = \sqrt{3(1-\nu^2)}/2\sqrt{R/h}$ and $A_{00}, A_{01}, A_{10}, A_{11}, B_{01}, B_{11}, B_{21}, A_{22}, A_{32}$ and B_{43} are constants of integration and $\ln \gamma = 0.57722$ are Euler's constant. Though we use Eq. (4.4) in the following to get a more exact solution, we may use the method in [6] to solve Eq. (4.2). Corresponding to Eqs. (4.4a,b), the solution of Eq. (2.6b) has the form

$$\chi = C_1 K_2(\eta \xi^2 \rho) \sin 2\theta + C_2 K_4(\eta \xi^2 \rho) \sin 4\theta \quad (4.5)$$

with $\eta = 4\sqrt{10/3(1-\nu^2)}$.

We here limit ourselves to a consideration of problems with wall thickness not too thick. By the analysis of order-of-magnitude, we know that the expression of V can be simplified into

$$V = W + AD \nabla^2 W$$

then we have expressins for stress resultants and stress couples

$$\left. \begin{aligned} N_r = & \frac{Eh}{2\alpha^2 R} \left(\frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \theta^2} \right) \text{Im}\sigma, \quad N_\theta = \frac{Eh}{2\alpha^2 R} \frac{\partial^2}{\partial \rho^2} \text{Im}\sigma \\ N_{r\theta} = & -\frac{Eh}{2\alpha^2 R} \frac{\partial}{\partial \rho} \left(\frac{1}{\rho} \frac{\partial}{\partial \theta} \right) \text{Im}\sigma \end{aligned} \right\} \quad (4.6a, b, c)$$

$$Q_r = -\frac{D}{R^3} \frac{\partial}{\partial \rho} \nabla^2 V + \frac{1}{R} \frac{\partial \chi}{\partial \theta}, \quad Q_\theta = -\frac{D}{R^3} \frac{\partial}{\partial \theta} \nabla^2 V - \frac{1}{R} \frac{\partial \chi}{\partial \rho} \quad (4.7a, b)$$

$$\begin{aligned} M_r = & -\frac{D}{R^2} \left(\frac{\partial^2}{\partial \rho^2} + \nu \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{\nu}{\rho^2} \frac{\partial^2}{\partial \theta^2} \right) \left(1 + \frac{AD \nabla^2}{R^2} \right) \text{Re}\sigma \\ & + \frac{AD}{R^2} (1-\nu) \frac{\partial}{\partial \rho} \left(\frac{1}{\rho} \frac{\partial}{\partial \theta} \right) \chi \end{aligned} \quad (4.8a)$$

$$\begin{aligned} M_\theta = & -\frac{D}{R^2} \left(\frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \theta^2} + \nu \frac{\partial^2}{\partial \rho^2} \right) \left(1 + \frac{AD \nabla^2}{R^2} \right) \text{Re}\sigma \\ & - \frac{AD}{R^2} (1-\nu) \frac{\partial}{\partial \rho} \left(\frac{1}{\rho} \frac{\partial}{\partial \theta} \right) \chi \end{aligned} \quad (4.8b)$$

$$\begin{aligned}
 M_{\rho\theta} = & -\frac{D}{R^2}(1-\nu)\frac{\partial}{\partial\rho}\left(\frac{1}{\rho}\frac{\partial}{\partial\theta}\right)\left(1+\frac{AD}{R^2}\nabla^2\right)\text{Re}\sigma \\
 & +\frac{(1-\nu)AD}{R^2}\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\theta^2}\right)\chi-\chi
 \end{aligned} \quad (4.8c)$$

where $\alpha=2\xi$

V. Stress Concentration Factors for Circular Cylindrical Shells with a Small Hole under Uniform Pressure

Given a circular cylindrical shell sealed at its two ends, in which there is uniform pressure, all stress resultants and stress couples are zero except the following

$$N_s=PR/2, \quad N_\varphi=PR$$

In coordinate system (ρ, θ) , we have

$$N_\rho=\frac{3}{4}PR-\frac{1}{4}PR\cos 2\theta, \quad N_\theta=\frac{3}{4}PR+\frac{1}{4}PR\cos 2\theta, \quad N_{\rho\theta}=\frac{1}{4}PR\sin 2\theta \quad (5.1a, b, c)$$

If there is a small circular hole on the shell, the state of stress is composed of the stress resultants expressed by Eqs. (5.1a, b, c) and the variation of stress produced by the presence of the hole.

The conditions of vanishing membrane stress resultants at the edge of the circular hole are of the homogeneous form

$$\rho=\rho_0, \quad N_\rho=N_{\rho\theta}=0 \quad (5.2)$$

An introduction of results of superposing Eqs. (4.6a, c) and Eqs. (5.1a, c) into Eq (5.2) gives a system of linear equations from which we obtain the following constants.

$$\left. \begin{aligned}
 A_{00} &= \frac{3\pi PR^2 \rho_0^3 \xi^2}{Eh}, \quad A_{10} = \frac{-2\pi PR^2 \rho_0^3 \xi^2}{Eh}, \quad B_{21} = \frac{\pi PR^2 \rho_0^4 \xi^2}{Eh} \\
 A_{01} &= \frac{2\pi^2 PR^2 \rho_0^4 \xi^2}{Eh}, \quad A_{11} = \frac{-5\pi^2 PR^2 \rho_0^4 \xi^2}{2Eh}, \quad B_{22} = \frac{5\pi^2 PR^2 \rho_0^4 \xi^2}{8Eh}
 \end{aligned} \right\} \quad (5.3)$$

Introducing the above constants into Eq. (4.6b), we get N_θ and the membrane stress resultants concentration factor

$$K_m = \frac{N_\theta}{N_\varphi} = \frac{5}{2} + \frac{9\pi \rho_0^3 \xi^2}{4} \quad (5.4)$$

This concentration factor is consistent with the results in [3].

The conditions of plate stress resultants and couples are of the form

$$\rho=\rho_0, \quad M_\rho=M_{\rho\theta}=0, \quad Q_\rho=-PR\rho_0/2 \quad (5.5)$$

Making an introduction of Eqs. (4.8a, c) and (4.7) into Eq. (5.5), we can obtain B_{11} and A_{32} .

$$\frac{B_{11}}{\rho_0^2} = \frac{1}{4f_s + 2(1+\nu)} \left\{ \left[\frac{71}{3} - \frac{11}{3}\nu + 20(1-\nu) \ln \frac{\xi \rho_0 \gamma}{\sqrt{2}} \right] \frac{a^2}{h} - \frac{8(3-\nu)}{1-\nu} h \right. \\ \left. - \frac{16fh}{1-\nu} \right\} \frac{\pi P \xi^2}{\pi} \quad (5.6a)$$

$$\frac{A_{32}}{\rho_0^2} = \frac{1}{12(1+\nu) + 24g} \left[g(16AD - a^2) - 8(1-2\nu)AD \right] \frac{\pi P \xi^2}{Eh} \quad (5.6b)$$

In these two expressions the functions f_s and g are given as follows

$$f_s = 1 - \frac{2}{\lambda} \frac{K_1(\lambda)}{K_2(\lambda)}, \quad g = 1 - \frac{6}{\lambda} \frac{K_3(\lambda)}{K_4(\lambda)}$$

At the edge of the hole, we obtain a convenient expression of $M_{\theta\theta}$ by making use of Eqs. (4.8a,b), in conjunction with one of the boundary conditions in Eq. (5.5), so as to have

$$M_{\theta} = -\frac{D}{R^2} (1+\nu) \nabla^2 \left(1 + \frac{AD}{R^2} \nabla^2 \right) \text{Re} \sigma |_{\rho=\rho_0} \quad (5.7)$$

With Eqs. (5.7), (5.3), and (5.6a,b), we have the stress couple concentration factor for the problem

$$K_s = \frac{6M_{\theta}}{h^2} / \frac{PR}{h} \\ = -\frac{3(1+\nu)a^2}{16Rh} \left\{ 9 + 16 \ln \frac{\xi \rho_0 \gamma}{\sqrt{2}} + \left[10 - \frac{1}{2f+1+\nu} \left(\frac{71}{3} - \frac{11}{3}\nu + 20(1-\nu) \ln \frac{\xi \rho_0 \gamma}{\sqrt{2}} \right. \right. \right. \\ \left. \left. - \frac{8(3-\nu)}{1-\nu} \frac{h^2}{a^2} - \frac{16f}{1-\nu} \frac{h^2}{a^2} \right] \cos 2\theta + \left[1 + \frac{2}{1+\nu+2g} \left(g \left(\frac{16AD}{a^2} - 1 \right) - 1 \right. \right. \right. \\ \left. \left. - 8(1-2\nu) \frac{AD}{a^2} \right) \right] \cos 4\theta \right\} + \frac{3(1+\nu)}{10(1-\nu)} \frac{h}{R} (5 \cos 2\theta + 4 \cos 4\theta) \quad (5.8)$$

Setting $\nu = 0.3$ and $\theta = 0$ or $\theta = \pi/2$, we have from the above equation

$$(K_s)_{\theta=0} = \frac{-3.9a^2}{16Rh} \left\{ 16.6172 + 16 \ln \rho_0 \sqrt{\frac{R}{h}} - \frac{1}{2f+1.3} \left[19.6067 + 16 \ln \rho_0 \sqrt{\frac{R}{h}} \right. \right. \\ \left. \left. - (30.8571 + 22.8571f) \left(\frac{h}{a} \right)^2 \right] + \frac{2}{1.3+2g} \left[g \left(4.57143 \left(\frac{h}{a} \right)^2 - 1 \right) - 1 \right. \right. \\ \left. \left. - 0.914286 \left(\frac{h}{a} \right)^2 \right] - 20.571 \left(\frac{h}{a} \right)^2 \right\} \quad (5.9a)$$

$$(K_s)_{\theta=\pi/2} = \frac{-3.9a^2}{16Rh} \left\{ -3.3828 + 16 \ln \rho_0 \sqrt{\frac{R}{h}} + \frac{1}{2f+1.3} \left[19.6067 \right. \right. \\ \left. \left. + 16 \ln \rho_0 \sqrt{\frac{R}{h}} - (30.8571 + 22.8571f) \left(\frac{h}{a} \right)^2 \right] \right. \\ \left. + \frac{2}{1.3+2g} \left[g \left(4.57143 \left(\frac{h}{a} \right)^2 - 1 \right) - 1 - 0.914286 \left(\frac{h}{a} \right)^2 \right] \right. \\ \left. + 0.22857 \frac{10h^2}{a^2} \right\} \quad (5.9b)$$

If $a \gg h$, that is to say, $h/a \rightarrow 0$, Eqs. (5.9a,b) become

$$\left. \begin{aligned} (K_b)_{\theta=0} &= -\frac{a^2}{Rh} \left(2.30676 + 2.86592 \ln \rho_0 \sqrt{\frac{R}{h}} \right) \\ (K_b)_{\theta=\pi/2} &= -\frac{a^2}{Rh} \left(0.32821 + 4.93409 \ln \rho_0 \sqrt{\frac{R}{h}} \right) \end{aligned} \right\} \quad (5.10a,b)$$

These results are consistent with the results obtained by the theory of thin shallow shells in [3].

The results calculated with Eq. (5.9) are shown in Fig. 3. It is apparent that there is a great difference between the results obtained in this paper and the results in [3], if λ is small, and the difference vanishes if λ is large.

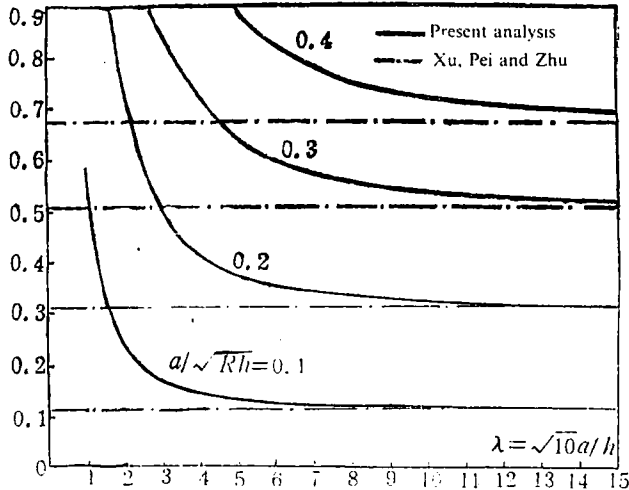


Fig. 3 Values of the bending stress concentration factor K_b ,
for $\theta = \pi/2$ and $\theta = 0.3$

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