

## THE MATHEMATICAL MODEL OF BUBBLE GROUP NOISE

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### Abstract

*In this paper the mathematical model of bubble group noise are introduced under the arbitrary conditions by using the method of Euler. The calculation indicates that the simulation results consist with the measured value.*

**Key words** cavitation, cavitation noise, mathematical model

### I. Noise at Collapse of an Isolated Bubble

The noise only has comparative big value at collapse period. By Ref. [1], we have:  
The radial velocity of bubble-wall of bubble at collapse

$$\dot{R} = \frac{dR}{dt} = \left\{ \frac{2}{3\rho} (p_{\infty} - p_v) \left[ \left( \frac{R_0}{R} \right)^3 - 1 \right] - \frac{2p_1}{3(1-\gamma)\rho} \left[ \left( \frac{R_0}{R} \right)^3 - \left( \frac{R_0}{R} \right)^{3\gamma} \right] + \frac{2\sigma}{\rho R} \left[ \left( \frac{R_0}{R} \right)^2 - 1 \right] \right\}^{\frac{1}{2}} \quad (1.1)$$

The sound pressure amplitude at the point Q with the radial distance  $r$  produced by an isolated bubble at collapse

$$p(R, r) = \frac{\sigma}{r} \left[ \left( \frac{R_0}{R} \right)^2 - 3 \right] + \frac{(p_{\infty} - p_v)}{r} \left\{ \frac{4R}{3} \left[ \left( \frac{R_0}{R} \right)^3 - 1 \right] - \frac{R_0^3}{R^2} \right\} - \frac{p_1}{(1-\gamma)r} \left\{ \frac{4R}{3} \left[ \left( \frac{R_0}{R} \right)^3 - \left( \frac{R_0}{R} \right)^{3\gamma} \right] - \frac{R_0^3}{R^2} + \frac{\gamma R_0^{3\gamma}}{R^{(3\gamma-1)}} \right\} \quad (1.2)$$

where  $R$ —radius of bubble at collapse;  $p_{\infty}$ —pressure at infinity;  $p_v = p_v(T)$ —vapour pressure in bubble;  $\sigma = \sigma(T)$ —surface tension of liquid;  $\gamma$ —gas constant (adiabatic) of air;  $p_1$ —gas pressure in bubble at initial moment of collapse;  $R_0$ —initial radius of bubble at collapse;  $T$ —temperature of liquid;  $\rho$ —density of liquid;  $t$ —time.

Substituting (1.1) into (1.2), we obtain the sound pressure amplitude of an isolated bubble at collapse  $p(t, r)$  (See Fig. 1).

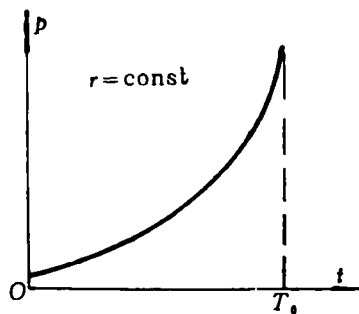


Fig. 1 The sound pressure amplitude of isolated bubble at collapse

## II. The Mathematical Model of Bubble Group Noise

In practice, the bubbles always appear in the form of the bubble group at the surface of body. The collapse time  $0 \sim T_0$  correspond to the distance  $0 \sim x_0$  on the surface of body for the any given body (See Fig. 2), where 0 is the collapse origin of bubble,  $x_0$  is the collapse terminal point of bubble ( $x_0$  correspond to the distance of bubble remove in  $T_0$ ).

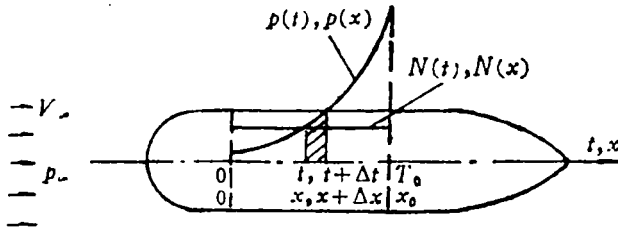


Fig. 2 The distribution of bubbles and its sound pressure

In general, the form of body, velocity and pressure at infinity are all permanent, therefore the flow field can be considered to be steady flow.

Suppose the number of bubbles produced on surface of body per unit time equals  $N$  (it is the number of bubble entering into the region of collapse per unit time). Under the steady conditions,  $N$  is a constant, its distribution along the  $x$  is uniform and the curve of  $p(x)$  is steady. Thus, the whole bubble group are steady uniform distribution from 0 to  $x_0$ . The number of bubbles in the  $\Delta x$  at the  $x$  is  $N\Delta t = N/V \cdot \Delta x$  (where  $V$  is the velocity of liquid at  $x$ ). The sound pressure amplitude at  $Q$  produced by any bubble in  $\Delta x$  is the sound pressure amplitude corresponding to  $x$ .

At any moment, the sound pressure amplitude at  $Q$  produced by bubble group is a constant that equals a sum of the sound pressure amplitude produced by each of the bubble in the  $0 \sim x_0$  region (See Fig. 3).

Thus

$$P_a = \int_0^{x_0} N/V \cdot p(x) dx = \int_0^{T_0} N p(t) dt \quad (2.1)$$

The sound pressure at  $Q$  can be expressed as

$$P(t) = P_a \exp[i\omega_0 t] \quad (2.2)$$

where  $\omega_0$ —the center angular frequency of sound pressure at  $Q$ .

By doing Fourier transform to the  $P(t)$ , we can find the amplitude frequency spectrum of bubble group noise at  $Q$  as (See Fig. 4)

$$A(\omega) = \left| \frac{P_a}{(\omega - \omega_0)} \right| \quad (2.3)$$

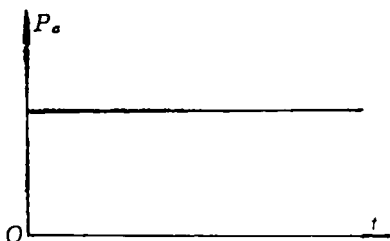


Fig. 3 The sound pressure amplitude of bubble group at collapses

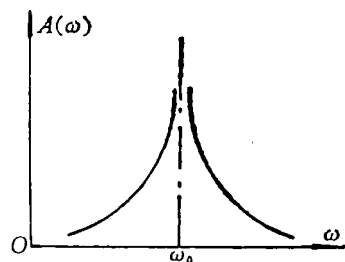


Fig. 4 The amplitude frequency spectrum of bubble group noise

### III. Discussion of Model Parameter

It is necessary to define  $p(t)$ ,  $N$ ,  $T_0$  and  $\omega_0$  in the above model. Now, we have a discussion on that.

1. It can be found from (1.2), the function  $p(t)$  is dependent on the parameters  $R_0$ ,  $p_\infty$  and  $T$ .

$R_0$ —initial radius of bubble at collapse. Under the constant pressure field, the value of  $R_0$  is dependent on the radius of gas nucleus. In natural water, the radius of gas nucleus is not equal to each other. In principle, the initial radius  $R_0$  of each bubble and its  $p(t)$  are not equal to each other. But the calculation indicate that the difference of gas nucleus scale can be omitted, because only at the initial stage of collapse dose it show the remarkable effect. The noise only has comparative big value at late period of collapse. Therefore, it is considered that the sound pressure amplitude  $p(t)$  of each bubble at collapse is equal to each other.

$p_\infty$ —pressure at infinity. The  $p_\infty$  is related to velocity of collapse, sound pressure and  $M^2$ . Its value can be obtained by measured value.

$T$ —temperature of liquid. It has influence on sound pressure through the medium of surface tension of liquid  $\sigma$  and vapour pressure  $p_v$ . When the change of temperature is small, its effect can be omitted.

2. The value of  $N$  equals the number of efficient gas nucleus entering into cavitation range per unit time. It is dependent on the density function of nuclei  $N(R)^{[1]}$ , velocity  $V$  and the cross-section area of cavitation range. Its value can be obtained by measured value.

3. It is generally believed that the minimum radius of bubble at collapse is about 2–10 per cent of the initial radius  $R_0$ . According to this conclusion, the value of  $T_0$  can be obtained from (1.1).

4. At present, the value of  $\omega_0$  can not be obtained by calculation. We can make an estimate of  $\omega_0$  on the basis of a large number of measured values.

### IV. Exemplification

In Fig. 5 it is shown that cavitation noise frequency spectrum curve of the revolution body measured in the water tunnel at NPU<sup>[4]</sup>. It is seen from Fig. 5 that the  $\omega_0 \approx 50\text{kHz}$ .

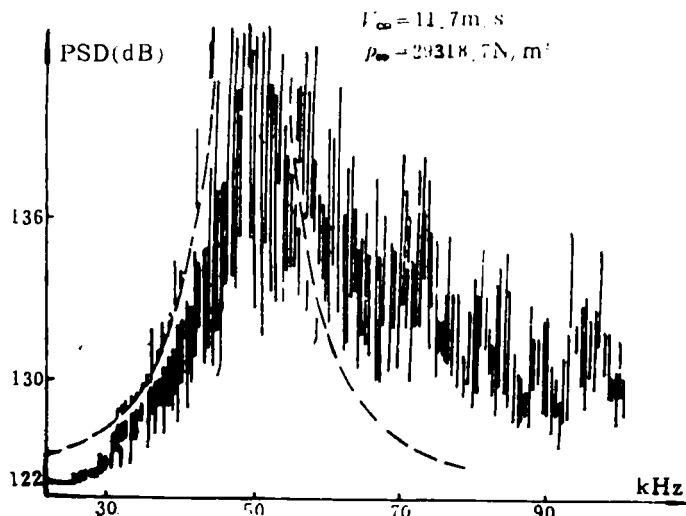


Fig. 5 The cavitation noise frequency spectrum curve of the revolution body

According to the measured values of flow field of the revolution body experiment, using the mathematical model presented here, the calculation is performed. Its results are shown with the dotted line in Fig. 5. It is obvious from Fig. 5 that simulation results consist with the measured values.

In reality, the cavitation noise is random, because of the impulse of the flow field, the randomness of bubble size and the trajectory of the bubble movement. It is considered that the mathematical model presented here is merely a model of the expectation value.

## References

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