

THE SOLUTION FOR TRANSIENT TWO-PHASE FLOW BY SPLIT FLUX VECTOR METHOD

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Abstract

In this paper the transient two-phase flow equations and their eigenvalues are first introduced. The flux vector is then split into subvectors which just contain a specially signed eigenvalue. Using one-sided spatial difference operators finite difference equations and their solutions are obtained. Finally comparison with experiment shows the predicted results produce good agreement with experimental data.

Key words two-phase flow, vector flux, split flux vector method

Nomenclature

c_p J/kg $^{\circ}$ C	specific heat at constant pressure	T $^{\circ}$ C, K	temperature
c m/s	sonic velocity	t s	time
d m	diameter of tube	u m/s	velocity
f	friction coefficient	V_g m 3 /kg	specific volume of saturated vapour
G kg/m 2 s	mass velocity	V_f m 3 /kg	specific volume of saturated liquid
g m/s 2	gravitational acceleration	Z m	length
h_f J/kg	specific enthalpy of saturated liquid	α	void fraction
h_g J/kg	specific enthalpy of saturated vapour	β m 3 .K/J	property relationship
h_{gf} J/kg	enthalpy of evaporation	λ	eigenvalue
P N/m 2	pressure	ρ kg/m 3	density
		ρ_m kg/m 3	mean density

I. Introduction

As the size of steam electric generating plant unit increases, and the capacities of the feedwater pumps increase, the safe operation for the feedwater pumps becomes increasingly important. The safe operation for the feedwater pumps depends either on the vaporization which has been studied a lot or on the degree of vaporization which has never been seen before. The light vaporization only causes the cavitation on the vane but the large vaporization will cause feedwater stop immediately. For example, after the slip of turbine load, the transient two-phase flow has been formed at the downcomer pipe when the deaerator is rapidly depressurized.

It is dangerous that bubble front reaches the suction of pumps so the problem of transient two-phase flow is very important for the safe operation of the feed-water pumps.

The transient two-phase flow equations are hyperbolic equations. The closed form solution of nonlinear hyperbolic partial differential equations does not exist for general cases. In order to obtain solutions to such equations, numerical methods are often required.

The method of characteristic finite difference is often used to solve hyper-bolic PDE's. Many authors^[1-3] used this method to solve these equations in both single phase and two-phase flows. But for subsonic flow, the eigenvalues of the equations are both positive and negative. So using one side spatial difference always produces instability. However, the one side spatial difference can increase the numerical efficiency.

Gino Moreth^[4] developed the λ scheme for integrating the Euler equations of gas dynamics. The scheme considered the wave direction of propagation and used the one side spatial difference. Recently, some authors^[5-6] suggested splitting the flux vector into subvectors so that each subvector is associated with a special eigenvalue and used this method to solve problems in single phase flow. But using split flux vector scheme in transient two-phase flow has never been seen before.

In this paper the flux vector split method to solve transient two-phase flow has been used. Comparison with experiment shows this method is a good scheme.

II. Transient Two Phase Flow Equations and Eigenvalues

In the absence of heat input and of slip thermal equilibrium with one-dimension flow the contral differential equations are^[1]

$$[A] \frac{\partial [W]}{\partial t} + [B] \frac{\partial [W]}{\partial z} = [C] \quad (2.1)$$

where

$$[W] = \begin{bmatrix} G \\ p \\ \alpha \end{bmatrix} \quad (2.2)$$

$$[A] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -c^2 \rho_{gf} \\ 0 & 0 & 1 \end{bmatrix} \quad (2.3)$$

$$[B] = \begin{bmatrix} \frac{2G}{\rho_m} & 1 & -\frac{G^2 \rho_{gf}}{\rho_m^2} \\ 0 & \frac{G}{\rho_m} & -\frac{c^2 G \rho_{gf}}{\rho_m} \\ \frac{1}{\rho_{gf}} & 0 & 0 \end{bmatrix} \quad (2.4)$$

$$[C] = \begin{bmatrix} g \rho_m - \frac{2fG^2}{d \rho_m} \\ \frac{2fG^3}{d \beta \rho_m^2 (\rho c_p)_m} \\ 0 \end{bmatrix} \quad (2.5)$$

c is the velocity of sound in the homogeneous equilibrium mixture given by

$$c^2 = \frac{h_{g,f} \rho_f \rho_g}{c_{p,f} \beta \rho_m^2 \rho_{f,g}} \quad (2.6)$$

$$\beta = \frac{T_f}{h_{g,f}} (V_g - V_f) \quad (2.7)$$

$$\rho_m = \alpha \rho_g + (1 - \alpha) \rho_f \quad (2.8)$$

Equation (2.1) can be expressed in a simpler form by multiplying it throughout by $[A]^{-1}$

$$\frac{\partial[W]}{\partial t} + [D] \frac{\partial[W]}{\partial z} = [E] \quad (2.9)$$

where

$$[D] = \begin{bmatrix} \frac{2G}{\rho_m} & 1 & -\frac{G^2 \rho_{g,f}}{\rho_m^2} \\ c^2 & \frac{G}{\rho_m} & -\frac{c^2 G \rho_{g,f}}{\rho_m} \\ \frac{1}{\rho_{g,f}} & 0 & 0 \end{bmatrix} \quad (2.10)$$

$$[E] = \begin{bmatrix} g \rho_m - \frac{2fG^2}{d \rho_m} \\ \frac{2fG^3}{d \rho_m^2 \beta (\rho c_p)_m} \\ 0 \end{bmatrix} \quad (2.11)$$

Equation (2.8) is a system of hyperbolic equations if the all eigenvalues are real. In order to find the eigenvalues of system we write

$$|[D] - \lambda[I]| = 0 \quad (2.12)$$

From (2.12) the eigenvalues of the equation are

$$\lambda_1 = \frac{G}{\rho_m} + c, \quad \lambda_2 = \frac{G}{\rho_m} - c, \quad \lambda_3 = \frac{G}{\rho_m} \quad (2.13)$$

where

$$u = G / \rho_m$$

is the velocity of fluid. If $|u| < c$ for the subsonic flow and the eigenvalues are mixed signs.

III. Compatibility Equations and Boundary Conditions

To solve equation (2.9), for the initial and boundary conditions in our case, generally, we need know mass velocity G , void fraction α and pressure value of both inlet and outlet of downcomer pipe. But for hyperbolic equation some of these boundary conditions can be determined by the initial value which propagates along the characteristic line, i.e. by the compatibility equations. The compatibility equations are derived below. Let L^i represent the left eigenvector of $[D]$ corresponding to λ_i . The eigenvectors of $[D]$ can be derived by writing

$$[L^i]\{[D]-\lambda_i[I]\}=0 \quad (3.1)$$

where

$$[L^i]=[L_1^i \quad L_2^i \quad L_3^i]$$

for $\lambda_1=G/\rho_m+c$, from (3.1), we obtain

$$[L^1]=[L_1^1 \quad L_2^1 \quad L_3^1] \begin{Bmatrix} \frac{G}{\rho_m}-c & 1 & -\frac{G^2\rho_{gf}}{\rho_m^2} \\ c^2 & -c & -\frac{c^2G\rho_{gf}}{\rho_m} \\ \frac{1}{\rho_{gf}} & 0 & -\frac{G}{\rho_m}-c \end{Bmatrix} = 0 \quad (3.2)$$

Solving equation (3.2), we obtain the eigenvector corresponding to λ_1

$$[L^1]=\begin{bmatrix} 1 & \frac{1}{c} & -\frac{\rho_{gf}G}{\rho_m} \end{bmatrix} \quad (3.3)$$

Similarly, the eigenvectors corresponding to λ_2 and λ_3 are

$$[L^2]=\begin{bmatrix} -1 & \frac{1}{c} & \frac{G\rho_{gf}}{\rho_m} \end{bmatrix} \quad (3.4)$$

$$[L^3]=\begin{bmatrix} 0 & -\frac{1}{c^2} & \rho_{gf} \end{bmatrix} \quad (3.5)$$

Now by premultiplying equation (2.9) by those eigenvectors, the compatibility equations can be obtained. Because

$$[L^i]\left\{\frac{\partial[W]}{\partial t}+[D]\frac{\partial[W]}{\partial z}\right\}=[L^i][E]$$

from (3.1), the above equation can be written

$$[L^i]\left\{\frac{\partial[W]}{\partial t}+\lambda_i\frac{\partial[W]}{\partial z}\right\}=[L^i][E] \quad (3.6)$$

So the compatibility equation along λ_1 is

$$\begin{aligned} & \frac{\partial G}{\partial t} + \frac{1}{c} \frac{\partial p}{\partial t} - \frac{\rho_{gf}G}{\rho_m} \frac{\partial \alpha}{\partial t} + \left(\frac{G}{\rho_m} + c\right) \frac{\partial G}{\partial z} + \left(\frac{G}{\rho_m} + c\right) \\ & \cdot \frac{1}{c} \frac{\partial p}{\partial z} - \left(\frac{G}{\rho_m} + c\right) \frac{\rho_{gf}G}{\rho_m} \frac{\partial \alpha}{\partial z} = g\rho_m - \frac{2fG^2}{d\rho_m} + \frac{2fG^3}{cd\rho_m^2\beta(\rho c_p)_m} \end{aligned} \quad (3.7a)$$

Similarly, the compatibility equations along λ_2 and λ_3 are

$$\begin{aligned} & -\frac{\partial G}{\partial t} + \frac{1}{c} \frac{\partial p}{\partial t} + \frac{\rho_{gf}G}{\rho_m} \frac{\partial \alpha}{\partial t} - \left(\frac{G}{\rho_m} - c\right) \frac{\partial G}{\partial z} + \left(\frac{G}{\rho_m} - c\right) \frac{1}{c} \frac{\partial p}{\partial z} \\ & + \left(\frac{G}{\rho_m} - c\right) \frac{\rho_{gf}G}{\rho_m} \frac{\partial \alpha}{\partial z} = -g\rho_m + \frac{2fG^2}{d\rho_m} + \frac{1}{c} \frac{2fG^3}{d\rho_m^2\beta(\rho c_p)_m} \end{aligned} \quad (3.7b)$$

$$-\frac{1}{c^2} \frac{\partial p}{\partial t} + \rho_{gf} \frac{\partial \alpha}{\partial t} - \frac{1}{c^2} \frac{G}{\rho_m} \frac{\partial p}{\partial z} + \rho_{gf} \frac{G}{\rho_m} \frac{\partial \alpha}{\partial z} = -\frac{1}{c^2} \frac{2fG^3}{d\rho_m^2 \beta(\rho c_p)_m} \quad (3.7c)$$

The Boundary Conditions

We know from equation (2.13) that at the inlet of downcomer pipe one characteristic line passing out of the grid region, thus the two imposed boundary conditions are required to leave one unknown to be calculated along the characteristic line. The two imposed boundary conditions are void fraction α and pressure P . Here the calculated boundary condition (by the compatibility equation (3.7b)) is the mass flux G at the top of the downcomer tube. It is also shown from equation (2.13) that there are two characteristic lines passing out of the region at the outlet of the downcomer pipe. Along the two characteristic lines two boundary conditions are calculated by equations (3.7a) and (3.7c). Here pressure P and void fraction α , and one unknown are imposed, where the mass velocity is given at the bottom of the downcomer pipe.

IV. Flux Vector Splitting and Finite Difference Equation

Now return to equation (2.9). Let

$$[T]^{-1} = \begin{Bmatrix} 1 & \frac{1}{c} & -\frac{\rho_{gf}G}{\rho_m} \\ -1 & \frac{1}{c} & \frac{\rho_{gf}G}{\rho_m} \\ 0 & -\frac{1}{c^2} & \rho_{gf} \end{Bmatrix}$$

From equation (2.9) we have the following equation

$$[T]^{-1} \frac{\partial [W]}{\partial t} + [A_D][T]^{-1} \frac{\partial [W]}{\partial z} = [T]^{-1} [E] \quad (4.1)$$

where $[A_D]$ is the diagonal matrix of the eigenvalue of (2.9)

$$[A_D] = \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{bmatrix} \quad (4.2)$$

Equation (4.1) can be transformed

$$\frac{\partial [W]}{\partial t} + [T][A_D][T]^{-1} \frac{\partial [W]}{\partial z} = [E] \quad (4.3)$$

Comparing equations (2.9) and (4.3), we obtain

$$[D] = [T][A_D][T]^{-1} \quad (4.4)$$

Now if λ_i , $[A_D]$ and $[D]$ can be split into positive and negative parts, namely

$$\lambda_i = \lambda_i^+ + \lambda_i^- \quad (4.5a)$$

$$[A_D] = [A_D]^+ + [A_D]^- \quad (4.5b)$$

$$[D] = [D]^+ + [D]^- \quad (4.5c)$$

we obtain

$$[D]^+ = [T][A_b]^+[T]^- \quad (4.6)$$

$$[D]^- = [T][A_b]^-[T]^- \quad (4.7)$$

where

$$\lambda_i^+ = \frac{\lambda_i + |\lambda_i|}{2} \quad \lambda_i^- = \frac{\lambda_i - |\lambda_i|}{2} \quad (4.8)$$

so the eigenvalues given by equation (2.13) are split according to equation (4.8) into

$$\lambda_1^+ = \frac{u+c+|u+c|}{2} \quad \lambda_1^- = \frac{u+c-|u+c|}{2} \quad (4.9a)$$

$$\lambda_2^+ = \frac{u-c+|u-c|}{2} \quad \lambda_2^- = \frac{u-c-|u-c|}{2} \quad (4.9b)$$

$$\lambda_3^+ = \frac{u+|u|}{2} \quad \lambda_3^- = \frac{u-|u|}{2} \quad (4.9c)$$

Finally, the corresponding subvectors $[D]^+$ and $[D]^-$ for the $0 \leq u \leq c$ are

$$[D]^+ = \begin{bmatrix} \frac{\lambda_1}{2} \left(1 + \frac{G}{c\rho_m} \right) \frac{\lambda_1}{2c} \left(1 + \frac{G}{c\rho_m} \right) - \frac{\lambda_3 G}{c^2 \rho_m} - \frac{\lambda_1 \rho_{gf} G}{2\rho_m} \left(1 + \frac{G}{c\rho_m} \right) + \frac{\lambda_3 \rho_{gf} G}{\rho_m} \\ \frac{\lambda_1 c}{2} & \frac{\lambda_1}{2} & -\frac{\lambda_1 \rho_{gf} G c}{2\rho_m} \\ \frac{\lambda_1}{2c\rho_{gf}} & \frac{\lambda_1}{2c^2 \rho_{gf}} - \frac{\lambda_3}{c^2 \rho_{gf}} & -\frac{\lambda_1 G}{2c\rho_m} + \lambda_3 \end{bmatrix} \quad (4.10a)$$

$$[D]^- = \begin{bmatrix} \frac{\lambda_2}{2} \left(1 - \frac{G}{c\rho_m} \right) & -\frac{\lambda_2}{2c} \left(1 - \frac{G}{c\rho_m} \right) & -\frac{\lambda_2 \rho_{gf} G}{2\rho_m} \left(1 - \frac{G}{c\rho_m} \right) \\ -\frac{\lambda_2 c}{2} & \frac{\lambda_2}{2} & \frac{\lambda_2 c \rho_{gf} G}{2\rho_m} \\ -\frac{\lambda_2}{2c\rho_{gf}} & \frac{\lambda_2}{2c^2 \rho_{gf}} & \frac{\lambda_2 G}{2c\rho_m} \end{bmatrix} \quad (4.10b)$$

Now consider equation (4.4). Equation (2.9) can be written

$$\frac{\partial[W]}{\partial t} + [D]^+ \frac{\partial[W]}{\partial z} + [D]^- \frac{\partial[W]}{\partial z} = [E] \quad (4.11)$$

If we used Fourier stability analysis^[8] we find that the second term of the left of equation (4.11) requires backward difference and the third term of the left of equation (4.11) requires forward difference for stability. Now if we denote a first order backward difference by ∇ and forward difference by Δ , (4.11) can be written as the following explicit finite difference equation

$$[W]_j^{t+1} = [W]_j^t + [E] \Delta t - \{ [D]^+ \nabla [W]_j^t + [D]^- \Delta [W]_j^t \} \frac{\Delta t}{\Delta z} \quad (4.12)$$

This equation (4.12) is stable if^[1]

$$|\lambda_{max}^{\pm}| \frac{\Delta t}{\Delta z} \leq 1 \quad (4.13)$$

V. Comparison Prediction with Experiment Data

In order to judge our theoretical analysis the downcomer pipe flow was chosen as numerical test. Referring to Figure 1, the downcomer pipe was divided into N -sections. The pressured and void fraction as imposed boundary condition at the top of pipe whilst the mass velocity was used at imposed boundary condition at the bottom as previously mentioned determine the use of compatibility equation. Figures 2 and 3 show the void fraction and pressure history with time at different section when $\Delta t = 0.001$, $\Delta z = 1$. The real line is the boundary condition at the inlet of pipe in Figures 2 and 3.

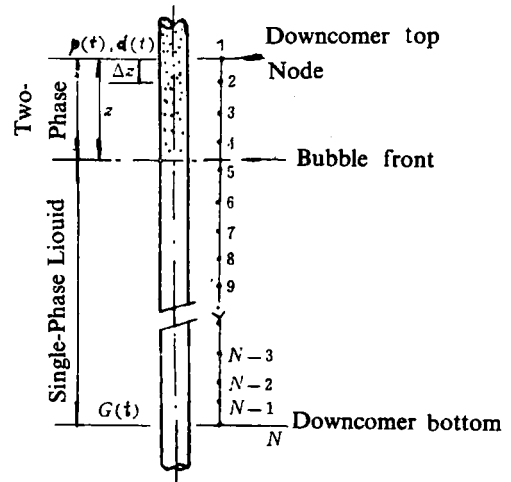
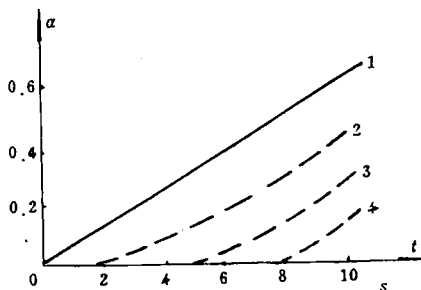
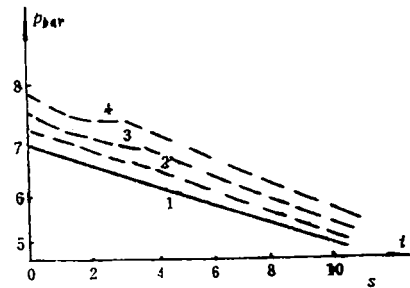


Fig. 1 Downcomer tube nodalisation



1. Inlet section 2. 3M from inlet
3. 6M from inlet 4. 10M from inlet

Fig. 2 Relation between α and t



1. Inlet section 2. 3M from inlet
3. 6M from inlet 4. 10M from inlet

Fig. 3 Relation between P and t

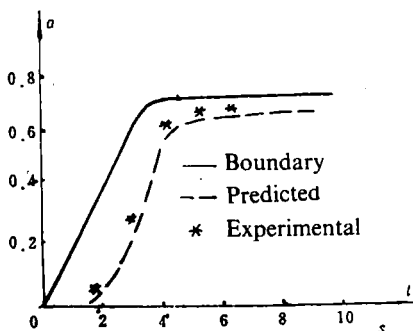


Fig. 4 Comparison of experimental and predicted results

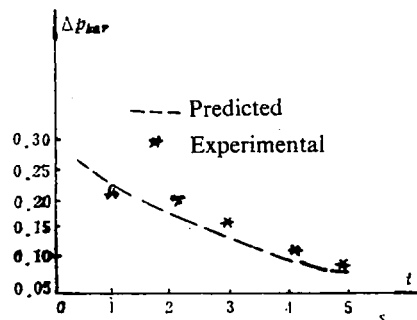


Fig. 5 Comparison of experimental and predicted results

Figures 4 and 5 show the comparison prediction with experiment data^[9]. Figure 4 shows the void fraction at the centre section against time. Figure 5 shows the pressure difference between the top and bottom of downcomer pipe against time. The real line is the boundary condition at inlet section in Figure 4. In Figure 4 the vain lines are of theroretical value and * are experiment data^[9]. From Figures 4 and 5 we know that the prediction results good agreement with experiment data.

VI. Conclusion

It is important for safe operation to solve the problem of fransient two-phase flow after slip turbine load.

The homogeneous equilibrium model seems to give a good model.

A transient two-phase flow equation can be solved by the splitting method which alows us of one sided spatial difference operator and this is important for solving transient two-phase flow equation.

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