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LARGE DEFLECTION PROBLEM OF A CLAMPED ELLIPTICAL PLATE SUBJECTED TO UNIFORM PRESSURE

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Abstract

In this paper, the perturbation solution of large deflection problem of clamped elliptical plate subjected to uniform pressure is given on the basis of the perturbation solution of large deflection problem of similar clamped circular plate (1948)[1]*, (1954)*[2]*. The analytical solution of this problem was obtained in 1957. However, due to social difficulties, these results have never been published. Nash and Cooley (1959)*[3] *published a brief note of similar nature, in which only the case* $\lambda=a/b=2$ *is given. In this paper, the analytical solution is given in detail up to the 2nd approximation. The numerical solutions are given for various Poisson ratios* $\nu=0.25$, *0.30, 0.35 and for various eccentricities* $\lambda=1, 2, 3, 4, 5$, *which can be used in the calculation of engineering designs.*

Key words elliptical plate, large deflection, perturbation method

I. Introduction

The large deflection problems of plates have interested applied mathematicians over a quite long period. In these problmes, Von Kármán's nonlinear differential equations need to be solved (1910)[4], (1940)[5]. Due to the difficulties in solving nonlinear differential equations, only very few problems have been solved. At the first, S.Way (1934)[6] gave the solution of circular clamped plate under uniform pressure in terms of infinite power series. Then, S. Levy (1942)[7] gave the double trigonometrical series solution for rectangular plate under uniformly distributed pressure. These two solutions are all very complicated and tedious so that they are hard to handle even in some important cases. Chien Wei-zang in 1948[1] and in 1954[2] treated the problems of large deflection of clamped circular plate under uniform pressure for various boundary conditions by the perturbation method, in which satisfactory results are obtained. The calculated displacement at the center of the plate and the calculated yield condition on the plate boundary agree very closely to the experimental results given by McPherson, Ramburg, and Levy (1942)[8]. From then on, Chien Wei-zang and Yeh Kai-yuan tried to solve the large deflection problem of rectangular plate under uniform pressure (1956)[9] by perturbation method; a little later, Nash and Cooley (1959)[3] tried the perturbation method to solve the large deflection problem of elliptical plate under uniform pressure, in which the case of axis ratios $\lambda=2$, $\nu=0.30$ has been calculated in numerical details. In this paper, the perturbed solutions of large deflection problem of clamped elliptical plate in various Poisson's ratios

$\nu = 0.25$, 0.30, 0.35 and various axis ratios $\lambda = 1$, 2, 3, 4, 5 are given for uniform pressure. Both in the first approximation, and in the second approximation, the perturbed equations are solved analytically.

II. Large Deflection Problem of Elliptical Plate under Uniform Pressure

Let us consider the large deflection problem of an elliptical plate with major and minor axes $2a$ and $2b$, thickness h under the action of transversal uniform load q (Fig.1). Let us denote the lateral deflection and the tensile displacements in x, y directions of the points $P(x, y)$ in the middle surface of the plate by w, u, v. The membrane stresses may be written as

$$\sigma_x = \frac{E}{1-\nu^2}\left\{\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \nu\left[\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right]\right\} \tag{2.1a}$$

$$\sigma_y = \frac{E}{1-\nu^2}\left\{\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + \nu\left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]\right\} \tag{2.1b}$$

$$\tau_{xy} = \frac{E}{2(1+\nu)}\left\{\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right\} \tag{2.1c}$$

in which E is Young's modulus, ν is Poisson's ratio. In the following calculations ν may be taken various values in 0.25–0.35.

Von Kármán's large deflection equations correspond to the following equations of equilibrium:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0, \qquad \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0 \tag{2.2a,b}$$

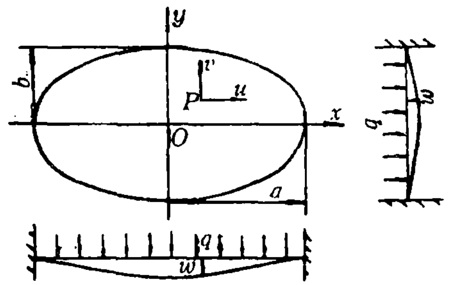

Fig. 1 Coordinates and displacements of elliptical plate

$$D\nabla^2\nabla^2 w = q + h\left(\sigma_x\frac{\partial^2 w}{\partial x^2} + \sigma_y\frac{\partial^2 w}{\partial y^2} + 2\tau_{xy}\frac{\partial^2 w}{\partial x\partial y}\right) \tag{2.2c}$$

in which $D = Eh^3/12(1-\nu^2)$, which is the fluxial rigidity of the plate, Substituting (2.1) into equation (2.2) gives:

$$2\frac{\partial^2 u}{\partial x^2} + (1-\nu)\frac{\partial^2 u}{\partial y^2} + (1+\nu)\frac{\partial^2 v}{\partial x\partial y} = -(1-\nu)\frac{\partial w}{\partial x}\nabla^2 w - \frac{1}{2}(1+\nu)\frac{\partial}{\partial x}\left[\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] \tag{2.3a}$$

$$2\frac{\partial v^2}{\partial y^2} + (1-\nu)\frac{\partial^2 v}{\partial x^2} + (1+\nu)\frac{\partial^2 u}{\partial x\partial y} = -(1-\nu)\frac{\partial w}{\partial y}\nabla^2 w - \frac{1}{2}(1+\nu)\frac{\partial}{\partial y}\left[\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] \tag{2.3b}$$

$$D\nabla^2\nabla^2 w = q + \frac{Eh}{1-\nu^2}\left\{\left(\frac{\partial u}{\partial x} + \nu\frac{\partial v}{\partial y}\right)\frac{\partial^2 w}{\partial x^2} + \left(\frac{\partial v}{\partial y} + \nu\frac{\partial u}{\partial x}\right)\frac{\partial^2 w}{\partial y^2} + (1-\nu)\cdot\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\frac{\partial^2 w}{\partial x\partial y}\right\} + \frac{Eh}{2(1-\nu^2)}\left\{\left[\left(\frac{\partial w}{\partial x}\right)^2 + \nu\left(\frac{\partial w}{\partial y}\right)^2\right]\frac{\partial^2 w}{\partial x^2} + \left[\left(\frac{\partial w}{\partial y}\right)^2 + \nu\left(\frac{\partial w}{\partial x}\right)^2\right]\frac{\partial^2 w}{\partial y^2} + 2(1-\nu)\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x\partial y}\right\} \tag{2.3c}$$

These equations may be non-dimensioned by introducing the following dimensionless quantities:

$$\left.\begin{aligned}&\lambda=\frac{a}{b},\quad \xi=\frac{x}{a},\quad \eta=\frac{y}{b},\quad Q=(1-\nu^2)\frac{a^4q}{h^4E}\\&U=\frac{au}{h^2},\quad V=\frac{bv}{h^2}=\frac{av}{\lambda h^2},\quad W=\frac{w}{h}\end{aligned}\right\}\tag{2.4}$$

Thus (2.3) may be written as:

$$2\frac{\partial^2U}{\partial\xi^2}+(1-\nu)\lambda^2\frac{\partial^2U}{\partial\eta^2}+(1+\nu)\lambda^2\frac{\partial^2V}{\partial\xi\partial\eta}$$
$$=-(1-\nu)\frac{\partial W}{\partial\xi}\left(\frac{\partial^2W}{\partial\xi^2}+\lambda^2\frac{\partial^2W}{\partial\eta^2}\right)-\frac{1}{2}(1+\nu)\frac{\partial}{\partial\xi}\left[\left(\frac{\partial W}{\partial\xi}\right)^2+\lambda^2\left(\frac{\partial W}{\partial\eta}\right)^2\right]\tag{2.5a}$$

$$2\lambda^2\frac{\partial^2V}{\partial\eta^2}+(1-\nu)\frac{\partial^2V}{\partial\xi^2}+(1+\nu)\frac{\partial^2U}{\partial\xi\partial\eta}$$
$$=-(1-\nu)\frac{\partial W}{\partial\eta}\left(\frac{\partial^2W}{\partial\xi^2}+\lambda^2\frac{\partial^2W}{\partial\eta^2}\right)-\frac{1}{2}(1+\nu)\frac{\partial}{\partial\eta}\left[\left(\frac{\partial W}{\partial\xi}\right)^2+\lambda^2\left(\frac{\partial W}{\partial\eta}\right)^2\right]\tag{2.5b}$$

$$\frac{\partial^4W}{\partial\xi^4}+2\lambda^2\frac{\partial^4W}{\partial\xi^2\partial\eta^2}+\lambda^4\frac{\partial^4W}{\partial\eta^4}-12Q=12\left\{\left(\frac{\partial U}{\partial\xi}+\nu\lambda^2\frac{\partial V}{\partial\eta}\right)\frac{\partial^2W}{\partial\xi^2}\right.$$
$$+\left(\lambda^2\frac{\partial V}{\partial\eta}+\nu\frac{\partial U}{\partial\xi}\right)\lambda^2\frac{\partial^2W}{\partial\eta^2}+\left(\frac{\partial U}{\partial\eta}+\frac{\partial V}{\partial\xi}\right)(1-\nu)\lambda^2\frac{\partial^2W}{\partial\eta\partial\xi}\Big\}+6\Big\{\left[\left(\frac{\partial W}{\partial\xi}\right)^2\right.$$
$$\left.+\nu\lambda^2\left(\frac{\partial W}{\partial\eta}\right)^2\right]\frac{\partial^2W}{\partial\xi^2}+\left[\lambda^2\left(\frac{\partial W}{\partial\eta}\right)^2+\nu\left(\frac{\partial W}{\partial\xi}\right)^2\right]\lambda^2\frac{\partial^2W}{\partial\eta^2}$$
$$\left.+2(1-\nu)\lambda^2\frac{\partial W}{\partial\xi}\frac{\partial W}{\partial\eta}\frac{\partial^2W}{\partial\xi\partial\eta}\right\}\tag{2.5c}$$

These equations will be solved with the boundary conditions:

$$W=U=V=\partial W/\partial n=0\,,\quad \text{on the boundary } \xi^2+\eta^2=1\tag{2.6}$$

in which n is the external normal direction on the boundary of elliptical plate.

III. Solution by Perturbation Method

According to the examples with most success in the perturbation method of the large deflection problems in circular plate (1948)[1], we take W_m, the dimensionless center deflection of elliptical plate, as the perturbing parameter,

$$W_m=W(0,\ 0)\tag{3.1}$$

Let us assume that U, V, W, Q in (2.5a, b, c) may be expanded into power series of W_m as follows:

$$U(\xi,\ \eta)=U_2(\xi,\ \eta)W_m^2+U_4(\xi,\ \eta)W_m^4+\cdots\tag{3.2a}$$

$$V(\xi,\ \eta)=V_2(\xi,\ \eta)W_m^2+V_4(\xi,\ \eta)W_m^4+\cdots\tag{3.2b}$$

$$W(\xi,\ \eta)=W_1(\xi,\ \eta)W_m+W_3(\xi,\ \eta)W_m^3+\cdots\tag{3.2c}$$

$$3Q/2(3+2\lambda^2+3\lambda^4)=a_1W_m+a_3W_m^3+\cdots\tag{3.2d}$$

Substituting (3.2a, b, c, d) into (2.5a, b, c) gives power series expansion of these equations in terms of W_m. These equations satisfy all the values of W_m. Hence, all the coefficients of W_m^n terms in these equations must vanish independently, from which we obtain the equations of various approximations for the determination of a_k, $U_k(\xi,\eta)$, $V_k(\xi,\eta)$, $W_k(\xi,\eta)$ successively.

Equations of the first approximation are given by the W_m terms in (2.5c) and W_m^2 terms in (2.5a,b) as follows. They are the equations for the determination of $W_1(\xi,\eta)$, a_1, $U_2(\xi,\eta)$ and $V_2(\xi,\eta)$:

$$\frac{\partial^4 W_1}{\partial\xi^4}+2\lambda^2\frac{\partial^4 W_1}{\partial\xi^2\partial\eta^2}+\lambda^4\frac{\partial^4 W_1}{\partial\eta^4}=8(3+2\lambda^2+3\lambda^4)a_1 \tag{3.3a}$$

$$2\frac{\partial^2 U_2}{\partial\xi^2}+(1-\nu)\lambda^2\frac{\partial^2 U_2}{\partial\eta^2}+(1+\nu)\lambda^2\frac{\partial^2 V_2}{\partial\xi\partial\eta}=-(1-\nu)\frac{\partial W_1}{\partial\xi}\left(\frac{\partial^2 W_1}{\partial\xi^2}+\lambda^2\frac{\partial^2 W_1}{\partial\eta^2}\right)-\frac{1}{2}(1+\nu)\frac{\partial}{\partial\xi}\left[\left(\frac{\partial W_1}{\partial\xi}\right)^2+\lambda^2\left(\frac{\partial W_1}{\partial\eta}\right)^2\right] \tag{3.3b}$$

$$2\lambda^2\frac{\partial^2 V_2}{\partial\eta^2}+(1-\nu)\frac{\partial^2 V_2}{\partial\xi^2}+(1+\nu)\frac{\partial^2 U_2}{\partial\xi\partial\eta}=-(1-\nu)\frac{\partial W_1}{\partial\eta}\left(\frac{\partial^2 W_1}{\partial\xi^2}+\lambda^2\frac{\partial^2 W_1}{\partial\eta^2}\right)-\frac{1}{2}(1+\nu)\frac{\partial}{\partial\eta}\left[\left(\frac{\partial W_1}{\partial\xi}\right)^2+\lambda^2\left(\frac{\partial W_1}{\partial\eta}\right)^2\right] \tag{3.3c}$$

We have from (3.1), (2.6) the following conditions for the solution of (3.3a, b, c):

$$W_1(0,0)=1 \tag{3.4a}$$

$$W_1(\xi,\eta)=U_2(\xi,\eta)=V_2(\xi,\eta)=\partial W_1(\xi,\eta)/\partial n=0,$$

$$\text{on the ellipse } \xi^2+\eta^2=1 \tag{3.4b,c,d,e}$$

Equations of the second approximation are given by the W_m^3 terms in (2.5c) and W_m^4 terms in (2.5a,b). Thus, we have the following equations for the determination of $W_3(\xi,\eta)$, a_3, $U_4(\xi,\eta)$, $V_4(\xi,\eta)$:

$$\begin{aligned}&\frac{\partial^4 W_3}{\partial\xi^4}+2\lambda^2\frac{\partial^4 W_3}{\partial\xi^2\partial\eta^2}+\lambda^4\frac{\partial^4 W_3}{\partial\eta^4}-8(3+2\lambda^2+3\lambda^4)a_3\\&=12\left\{\left(\frac{\partial U_2}{\partial\xi}+\nu\lambda^2\frac{\partial V_2}{\partial\eta}\right)\frac{\partial^2 W_1}{\partial\xi^2}+\left(\lambda^2\frac{\partial V_2}{\partial\eta}+\nu\frac{\partial U_2}{\partial\xi}\right)\lambda^2\frac{\partial^2 W_1}{\partial\eta^2}+\left(\frac{\partial U_2}{\partial\eta}+\frac{\partial V_2}{\partial\xi}\right)\right.\\&\left.\cdot(1-\nu)\lambda^2\frac{\partial^2 W_1}{\partial\xi\partial\eta}\right\}+6\left\{\left[\left(\frac{\partial W_1}{\partial\xi}\right)^2+\nu\lambda^2\left(\frac{\partial W_1}{\partial\eta}\right)^2\right]\frac{\partial^2 W_1}{\partial\xi^2}+\left[\lambda^2\left(\frac{\partial W_1}{\partial\eta}\right)^2\right.\right.\\&\left.\left.+\nu\left(\frac{\partial W_1}{\partial\xi}\right)^2\right]\lambda^2\frac{\partial^2 W_1}{\partial\eta^2}+2(1-\nu)\lambda^2\frac{\partial W_1}{\partial\xi}\frac{\partial W_1}{\partial\eta}\frac{\partial^2 W_1}{\partial\xi\partial\eta}\right\}\end{aligned} \tag{3.5a}$$

$$\begin{aligned}&2\frac{\partial^2 U_4}{\partial\xi^2}+(1-\nu)\lambda^2\frac{\partial^2 U_4}{\partial\eta^2}+(1+\nu)\lambda^2\frac{\partial^2 V_4}{\partial\xi\partial\eta}=-(1-\nu)\left\{\frac{\partial W_1}{\partial\xi}\left(\frac{\partial^2 W_3}{\partial\xi^2}+\lambda^2\frac{\partial^2 W_3}{\partial\eta^2}\right)\right.\\&\left.+\frac{\partial W_3}{\partial\xi}\left(\frac{\partial^2 W_1}{\partial\xi^2}+\lambda^2\frac{\partial^2 W_1}{\partial\eta^2}\right)\right\}-(1+\nu)\frac{\partial}{\partial\xi}\left[\frac{\partial W_1}{\partial\xi}\frac{\partial W_3}{\partial\xi}+\lambda^2\frac{\partial W_1}{\partial\eta}\frac{\partial W_3}{\partial\eta}\right]\end{aligned} \tag{3.5b}$$

$$\begin{aligned}&2\lambda^2\frac{\partial^2 V_4}{\partial\eta^2}+(1-\nu)\frac{\partial^2 V_4}{\partial\xi^2}+(1+\nu)\frac{\partial^2 U_4}{\partial\xi\partial\eta}=-(1-\nu)\left\{\frac{\partial W_1}{\partial\eta}\left(\frac{\partial^2 W_3}{\partial\xi^2}+\lambda^2\frac{\partial^2 W_3}{\partial\eta^2}\right)\right.\\&\left.+\frac{\partial W_3}{\partial\eta}\left(\frac{\partial^2 W_1}{\partial\xi^2}+\lambda^2\frac{\partial^2 W_1}{\partial\eta^2}\right)\right\}-(1+\nu)\frac{\partial}{\partial\eta}\left[\frac{\partial W_1}{\partial\xi}\frac{\partial W_3}{\partial\xi}+\lambda^2\frac{\partial W_1}{\partial\eta}\frac{\partial W_3}{\partial\eta}\right]\end{aligned} \tag{3.5c}$$

The boundary conditions for the solution of (3.5a, b, c) are

$$W_3(0,\ 0)=0 \tag{3.6a}$$

$$W_3(\xi,\ \eta)=U_4(\xi,\ \eta)=V_4(\xi,\ \eta)=\partial W_3(\xi,\ \eta)/\partial n=0$$

$$\text{on the ellipse } \xi^2+\eta^2=1 \tag{3.6b,c,d,e}$$

and so on. These equations can be solved successively. According to the experience in circular plate problems, the solution of the second approximation gives sufficient accurate solution for ordinary purpose.

IV. Solution of First Approximation

Let us solve (3.3a) with the boundary conditions (3.4a, b, e). The solution satisfying (3.4b, e) may be written as

$$W_1(\xi,\ \eta)=A_1^*(1-\xi^2-\eta^2)^2, \qquad A_1^*=\text{constant to be determined.} \tag{4.1}$$

Substituting this into (3.2a) gives

$$A_1^*=a_1 \tag{4.2}$$

Thus applying (4.1) to (3.4a), we obtain

$$A_1^*=1 \tag{4.3a}$$

from which we obtain the undetermined constant

$$a_1=1 \tag{4.3b}$$

and also the solution of (3.3a) under the conditions (3.4a, b, e) (1st approximation solution)

$$W_1(\xi,\ \eta)=(1-\xi^2-\eta^2)^2, \qquad a_1=1 \tag{4.4}$$

Substituting $W_1(\xi,\ \eta)$ from (4.4) into (3.3b, c), we obtain the system of two equations for the determination of $U_2(\xi,\ \eta)$ and $V_2(\xi,\ \eta)$.

$$2\frac{1}{\lambda^2}\frac{\partial^2 U_2}{\partial\xi^2}+(1-\nu)\frac{\partial^2 U_2}{\partial\eta^2}+(1+\nu)\frac{\partial^2 V_2}{\partial\xi\partial\eta}$$
$$=16\xi(1-\xi^2-\eta^2)\left\{\left[\frac{6}{\lambda^2}+(1-\nu)\right]\xi^2+\left[\frac{2}{\lambda^2}+5-\nu\right]\eta^2-\frac{2}{\lambda^2}-(1-\nu)\right\} \tag{4.5a}$$

$$2\lambda^2\frac{\partial^2 V_2}{\partial\eta^2}+(1-\nu)\frac{\partial^2 V_2}{\partial\xi^2}+(1+\nu)\frac{\partial^2 U_2}{\partial\xi\partial\eta}$$
$$=16\eta(1-\eta^2-\xi^2)\{[6\lambda^2+(1-\nu)]\eta^2+[2\lambda^2+5-\nu]\xi^2-2\lambda^2-(1-\nu)\} \tag{4.5b}$$

From (4.5a, b), it can be shown that, if U_2, V_2, ξ, η, $1/\lambda^2$ in (4.5a) are changed into V_2, U_2, η, ξ, λ^2, we readily obtain equation (4.5b). That is to say, if U_2 is a function of ξ, η, λ^2, ν, i.e.

$$U_2=f(\xi,\ \eta,\ \lambda^2,\ \nu) \tag{4.6a}$$

then V_2 must be the same function of η, ξ, $1/\lambda^2$, ν, i.e.

$$V_2=f(\eta,\ \xi,\ 1/\lambda^2,\ \nu) \tag{4.6b}$$

On the other hand, we have

$$V_2=g(\xi,\ \eta,\ \lambda^2,\ \nu), \qquad U_2=g(\eta,\ \xi,\ 1/\lambda^2,\ \nu) \tag{4.7a, b}$$

By the symmetrical properties and the boundary conditions (3.4c, d) of displacements, we may assume that the solutions of (4.5a, b) may take the following forms:

$$U_2(\xi,\ \eta)=\xi(1-\xi^2-\eta^2)\{A_1\xi^4+B_1\xi^2\eta^2+C_1\eta^4+D_1\xi^2+E_1\eta^2+F_1\} \tag{4.8a}$$

$$V_2(\xi,\ \eta)=\eta(1-\eta^2-\xi^2)\{A_2\eta^4+B_2\eta^2\xi^2+C_2\xi^4+D_2\eta^2+E_2\xi^2+F_2\} \tag{4.8b}$$

in which A_1, A_2, B_1, B_2, C_1, C_2, D_1, D_2, E_1, E_2, F_1, F_2 are functions of λ^2, ν to be determined. On the basis of (4.6a, b) or (4.7a, b), if A_1 is a function of λ^2, ν, then A_2 must be the same function of $1/\lambda^2$, ν. That is,

$$A_1=A_1(\lambda^2,\ \nu),\qquad A_2=A_1(1/\lambda^2,\ \nu) \tag{4.9a, b}$$

B_1 and B_2, C_1 and C_2, D_1 and D_2, E_1 and E_2, F_1 and F_2 have similar relations. It can be easily seen that (4.8a, b) satisfy the boundary conditions (3.4c, d). Substituting (4.8a, b) into (4.5a, b) gives

$$\begin{aligned}&\left[\frac{42}{\lambda^2}A_1+(1-\nu)(A_1+B_1)+3(1+\nu)C_2\right]\xi^4+\left[\frac{20}{\lambda^2}(A_1+B_1)+6(1-\nu)(B_1+C_1)\right.\\&\quad\left.+6(1+\nu)(B_2+C_2)\right]\xi^2\eta^2+\left[\frac{6}{\lambda^2}(B_1+C_1)+15(1-\nu)C_1+5(1+\nu)(A_2+B_2)\right]\eta^4\\&\quad+\left[\frac{20}{\lambda^2}(D_1-A_1)+(1-\nu)(D_1+E_1-B_1)+2(1+\nu)(E_2-C_2)\right]\xi^2\\&\quad+\left[\frac{6}{\lambda^2}(D_1+E_1-B_1)+6(1-\nu)(E_1-C_1)+3(1+\nu)(D_2+E_2-B_2)\right]\eta^2\\&\quad+\left[\frac{6}{\lambda^2}(F_1-D_1)+(1-\nu)(F_1-E_1)+(1+\nu)(F_2-E_2)\right]\\&=-8(1-\xi^2-\eta^2)\left\{\left[\frac{6}{\lambda^2}+1-\nu\right]\xi^2+\left[\frac{2}{\lambda^2}+5-\nu\right]\eta^2-\frac{2}{\lambda^2}-(1-\nu)\right\}\end{aligned} \tag{4.10a}$$

$$\begin{aligned}&[42\lambda^2A_2+(1-\nu)(A_2+B_2)+3(1+\nu)C_1]\eta^4+[20\lambda^2(A_2+B_2)+6(1-\nu)(B_2+C_2)\\&\quad+6(1+\nu)(B_1+C_1)]\xi^2\eta^2+[6\lambda^2(B_2+C_2)+15(1-\nu)C_2+5(1+\nu)(B_1+C_1)]\xi^4\\&\quad+[20\lambda^2(D_2-A_2)+(1-\nu)(D_2+E_2-B_2)+2(1+\nu)(E_1-C_1)]\eta^2\\&\quad+[6\lambda^2(D_2+E_2-B_2)+6(1-\nu)(E_2-C_2)+3(1+\nu)(D_1+E_1-B_1)]\xi^2\\&\quad+[6\lambda^2(F_2-D_2)+(1-\nu)(F_2-E_2)+(1+\nu)(F_1-E_1)]\\&=-8(1-\xi^2-\eta^2)\{[6\lambda^2+1-\nu]\eta^2+[2\lambda^2+5-\nu]\xi^2-2\lambda^2-(1-\nu)\}\end{aligned} \tag{4.10b}$$

(4.10a, b) are applicable to all the points in the region $\xi^2+\eta^2\leqslant 1$. Thus, the coefficients of the terms $\xi^4, \xi^2\eta^2$, η^4, ξ^2, η^2, 1 in the two sides of equation (4.10a) must be equal to each other. This gives

$$\frac{42}{\lambda^2}A_1+(1-\nu)(A_1+B_1)+3(1+\nu)C_2=8\left[\frac{6}{\lambda^2}+1-\nu\right] \tag{4.11a}$$

$$\frac{20}{\lambda^2}(A_1+B_1)+6(1-\nu)(B_1+C_1)+6(1+\nu)(B_2+C_2)=8\left[\frac{8}{\lambda^2}+2(3-\nu)\right] \tag{4.11b}$$

$$\frac{6}{\lambda^2}(B_1+C_1)+15(1-\nu)C_1+5(1+\nu)(A_2+B_2)=8\left[\frac{2}{\lambda^2}+5-\nu\right] \tag{4.11c}$$

$$\frac{20}{\lambda^2}(D_1-A_1)+(1-\nu)(D_1+E_1-B_1)+2(1+\nu)(E_2-C_2)=-8\left[\frac{8}{\lambda^2}+2(1-\nu)\right] \tag{4.11d}$$

$$\frac{6}{\lambda^2}(D_1+E_1-B_1)+6(1-\nu)(E_1-C_1)+3(1+\nu)(D_2+E_2-B_2)$$

$$=-8\left[\frac{4}{\lambda^2}+2(3-\nu)\right] \quad (4.11e)$$

$$\frac{6}{\lambda^2}(F_1-D_1)+(1-\nu)(F_1-E_1)+(1+\nu)(F_2-E_2)=8\left[\frac{2}{\lambda^2}+1-\nu\right] \quad (4.11f)$$

Similarly, the coefficients of terms η^4, $\eta^2\xi^2$, ξ^4, η^2, ξ^2, 1 in the two sides of equation (4.10b) must be equal to each other. This gives

$$42\lambda^2A_2+(1-\nu)(A_2+B_2)+3(1+\nu)C_1=8[6\lambda^2+1-\nu] \quad (4.12a)$$

$$20\lambda^2(A_2+B_2)+6(1-\nu)(B_2+C_2)+6(1+\nu)(B_1+C_1)=8[8\lambda^2+2(3-\nu)] \quad (4.12b)$$

$$6\lambda^2(B_2+C_2)+15(1-\nu)C_2+5(1+\nu)(A_1+B_1)=8[2\lambda^2+5-\nu] \quad (4.12c)$$

$$20\lambda^2(D_2-A_2)+(1-\nu)(D_2+E_2-B_2)+2(1+\nu)(E_1-C_1)=-8[8\lambda^2+2(1-\nu)] \quad (4.12d)$$

$$6\lambda^2(D_2+E_2-B_2)+6(1-\nu)(E_2-C_2)+3(1+\nu)(D_1+E_1-B_1)=-8[4\lambda^2+2(3-\nu)] \quad (4.12e)$$

$$6\lambda^2(F_2-D_2)+(1-\nu)(F_2-E_2)+(1+\nu)(F_1-E_1)=8[2\lambda^2+1-\nu] \quad (4.12f)$$

The above equations in (4.11) and (4.12) may be divided into two groups. (4.11a, b, c) and (4.12a, b, c) are the six equations for the determination of six unknowns A_1, B_1, C_1, A_2, B_2, C_2. When A_1, B_1, C_1, A_2, B_2, C_2 are determined, and substituting these results into (4.11d, e, f), (4.12d, e, f) also gives six independent equations for the determination of six unknowns D_1, E_1, F_1, D_2, E_2, F_2. Thus, all the unknown coefficients in (4.7a, b) can be determined, and therefore, equations (4.8a, b) represent the first approximation solution $U_2(\xi,\eta)$ and $V_2(\xi,\eta)$ of the functions $U(\xi,\eta)$ and $V(\xi,\eta)$.

Eliminating C_2 from (4.11a, b) gives the expression for B_2 in terms of A_1, B_1, C_1. Substituting this expression of B_2 into (4.11c) gives the expression for A_2 in terms of A_1, B_1, C_1. The expressions for A_2, B_2 obtained in the above calculation, together with the expression C_2 derived from (4.11a), from the following matrix equation.

$$15(1+\nu)\boldsymbol{\Phi}_2+\boldsymbol{\alpha}(1/\lambda^2)\boldsymbol{\Phi}_1=\mathbf{Q}_1(1/\lambda^2) \quad (4.13)$$

in which $\boldsymbol{\Phi}_1$, $\boldsymbol{\Phi}_2$, $\boldsymbol{\alpha}(1/\lambda^2)$, $\mathbf{Q}(1/\lambda^2)$ are respectively the following matrices, and $\boldsymbol{\alpha}(1/\lambda^2)$, $\mathbf{Q}_1(1/\lambda^2)$ are functions of $1/\lambda^2$:

$$\boldsymbol{\Phi}_1=\begin{Bmatrix} A_1 \\ B_1 \\ C_1 \end{Bmatrix}, \quad \boldsymbol{\Phi}_2=\begin{Bmatrix} A_2 \\ B_2 \\ C_2 \end{Bmatrix}, \quad \mathbf{Q}_1\left(\frac{1}{\lambda^2}\right)=\begin{Bmatrix} 128/\lambda^2+8(5-3\nu) \\ -80/\lambda^2+80 \\ 240/\lambda^2+40(1-\nu) \end{Bmatrix} \quad (4.14a,b,c)$$

$$\boldsymbol{\alpha}\left(\frac{1}{\lambda^2}\right)=\begin{bmatrix} 160/\lambda^2+5(1-\nu) & -32/\lambda^2-10(1-\nu) & 18/\lambda^2+30(1-\nu) \\ -160/\lambda^2-5(1-\nu) & 50/\lambda^2+10(1-\nu) & 15(1-\nu) \\ 210/\lambda^2+5(1-\nu) & 5(1-\nu) & 0 \end{bmatrix} \quad (4.14d)$$

Similarly, from (4.12a, b, c), after similar calculation, we obtain the following matrix equation:

$$15(1+\nu)\boldsymbol{\Phi}_1+\boldsymbol{\alpha}(\lambda^2)\boldsymbol{\Phi}_2=\mathbf{Q}_1(\lambda^2) \quad (4.15)$$

in which $\boldsymbol{\Phi}_1$, $\boldsymbol{\Phi}_2$ are given in (4:14a, b), and $\boldsymbol{\alpha}(\lambda^2)$, $\mathbf{Q}_1(\lambda^2)$ are similar to $\boldsymbol{\alpha}(1/\lambda^2)$, $\mathbf{Q}_1(1/\lambda^2)$ as shown in (4.14c, d), in which the variable $1/\lambda^2$ is substituted by λ^2.

From (4.13), (4.15), we get the solution of $\boldsymbol{\Phi}_1$, $\boldsymbol{\Phi}_2$:

$$\Phi_1=[225(1-\nu)^2\mathbf{I}-\alpha(\lambda^2)\alpha(1/\lambda^2)]^{-1}\{-\alpha(\lambda^2)\mathbf{Q}_1(1/\lambda^2)+15(1+\nu)\mathbf{Q}_1(\lambda^2)\} \tag{4.16a}$$

$$\Phi_2=[225(1+\nu)^2\mathbf{I}-\alpha(1/\lambda^2)\alpha(\lambda^2)]^{-1}\{-\alpha(1/\lambda^2)\mathbf{Q}_1(\lambda^2)+15(1+\nu)\mathbf{Q}_1(1/\lambda^2)\} \tag{4.16b}$$

in which $\mathbf{I}$ is a unitary matrix.

By means of (4.11a, b), the B_2, C_2 are eliminated by (4.11d, e, f), and finally the following matrix equation is obtained:

$$6(1+\nu)\Psi_2+\beta(1/\lambda^2)\Phi_1+\gamma(1/\lambda^2)\Psi_1=-\mathbf{Q}_2(1/\lambda^2) \tag{4.17}$$

in which Φ_1 can be found in (4.14a). Ψ_1, Ψ_2, $\mathbf{Q}_2(1/\lambda^2)$ are

$$\Psi_1=\begin{Bmatrix} D_1 \\ E_1 \\ F_1 \end{Bmatrix}, \quad \Psi_2=\begin{Bmatrix} D_2 \\ E_2 \\ F_2 \end{Bmatrix}, \quad \mathbf{Q}_2\left(\frac{1}{\lambda^2}\right)=\begin{Bmatrix} 32 \\ 96/\lambda^2+32(1-\nu) \\ -16(1-\nu) \end{Bmatrix} \tag{4.18a,b,c}$$

$\beta(1/\lambda^2)$, $\gamma(1/\lambda^2)$ are matrices in functions of $1/\lambda^2$:

$$\beta\left(\frac{1}{\lambda^2}\right)=\begin{bmatrix} -88/\lambda^2-4(1-\nu) & 8/\lambda^2+5(1-\nu) & -6(1-\nu) \\ 24/\lambda^2+2(1-\nu) & -(1-\nu) & 0 \\ 24/\lambda^2+2(1-\nu) & -(1-\nu) & 0 \end{bmatrix} \tag{4.18d}$$

$$\gamma\left(\frac{1}{\lambda^2}\right)=\begin{bmatrix} -48/\lambda^2-3(1-\nu) & 12/\lambda^2+9(1-\nu) & 0 \\ 60/\lambda^2+3(1-\nu) & 3(1-\nu) & 0 \\ 24/\lambda^2+3(1-\nu) & -3(1-\nu) & 36/\lambda^2+6(1-\nu) \end{bmatrix} \tag{4.18e}$$

Similarly, we obtain from (4.12d, e, f) the following equation:

$$6(1+\nu)\Psi_1+\beta(\lambda^2)\Phi_2+\gamma(\lambda^2)\Psi_2=-\mathbf{Q}_2(\lambda^2) \tag{4.19}$$

in which Φ_2, Ψ_1, Ψ_2 are shown respectively in (4.14b), (4.18a, b), and matrices $\beta(\lambda^2)$, $\gamma(\lambda^2)$, $\mathbf{Q}_2(\lambda^2)$ are shown in (4.18c, d, e) except the $1/\lambda^2$ in (4.18) changes into λ^2.

From (4.17), (4.19), we obtain Ψ_1, Ψ_2 as:

$$\begin{aligned}\Psi_1=&[36(1+\nu)^2\mathbf{I}-\gamma(\lambda^2)\gamma(1/\lambda^2)]^{-1}\{-6(1+\nu)\mathbf{Q}_2(\lambda^2)+\gamma(\lambda^2)\mathbf{Q}_2(1/\lambda^2)\\ &+6(1+\nu)\beta(\lambda^2)\Phi_2+\gamma(\lambda^2)\beta(1/\lambda^2)\Phi_1\}\end{aligned} \tag{4.20a}$$

$$\begin{aligned}\Psi_2=&[36(1+\nu)^2\mathbf{I}-\gamma(1/\lambda^2)\gamma(\lambda^2)]^{-1}\{-6(1+\nu)\mathbf{Q}_2(1/\lambda^2)+\gamma(1/\lambda^2)\mathbf{Q}_2(\lambda^2)\\ &+6(1+\nu)\beta(1/\lambda^2)\Phi_1+\gamma(1/\lambda^2)\beta(\lambda^2)\Phi_2\}\end{aligned} \tag{4.20b}$$

in which Φ_1, Φ_2 are given as in (4.16a, b). From (4.16a, b) and (4.20a, b) we can determine 12 unknowns A_1, B_1, C_1, A_2, B_2, C_2, D_1, E_1, F_1, D_2, E_2, F_2. Therefore, the first approximation solution of U, V, i.e. U_2, V_2 is obtained.

It should be noted that, if $\lambda^2=1/\lambda^2=1$, this elastic plate must be a circular plate. In this case, the undetermined coefficients have symmetrical properties, that is,

$$A_2=C_1=C_2=A_1, \qquad B_1=B_2=2A_1 \tag{4.21}$$

Thus, (4.11a, b, c) and (4.12a, b, c) are identical to each other, and can be reduced into the following common form:

$$A_1=(7-\nu)/6 \tag{4.22}$$

Hence, we have

$$A_1=A_2=C_1=C_2=(7-\nu)/6, \qquad B_1=B_2=(7-\nu)/3 \tag{4.23}$$

Substitute the result (4.23) into (4.11d, e, f) and (4.12d, e, f), and take $\lambda^2=1/\lambda^2=1$, and also,

$$D_1=E_1=D_2=E_2, \qquad F_1=F_2 \tag{4.24}$$

Then equations (4.11d, e) and (4.12d, e) are identical to each other, from which we get

$$D_1=E_1=D_2=E_2=-(13-3\nu)/6 \tag{4.25}$$

The third equations (4.11f), (4.12f) give:

$$F_1-D_1=3-\nu \tag{4.26}$$

Using the value of D_1 from (4.25), we have

$$F_1=F_2=(5-3\nu)/6 \tag{4.27}$$

Hence, in circular plate, $\lambda^2=1/\lambda^2=1$, (4.7a, b) may be written as:

$$\frac{U_2(\xi,\ \eta)}{\xi}=\frac{V_2(\xi,\ \eta)}{\eta}=\frac{1}{6}(1-\xi^2-\eta^2)\{(7-\nu)(1-\xi^2-\eta^2)^2-(1+\nu)(1-\xi^2-\eta^2)$$

$$-(1+\nu)\} \tag{4.28}$$

This is identical to the results given by Chien Wei-zang (1948)[1] for circular plate. Tables 1(A), 1(B), 1(C) are respectively the values of the coefficients in (4.8a, b) when ν takes various values 0.25, 0.30, 0.35. These results are calculated through formulas (4.16a, b) and (4.20a, b).

Table 1 (A) The values of coefficients for U_2, V_2 in (4.8a, b) when ν=0.25

λ	A_1	B_1	C_1	D_1	E_1	F_1
1	1.12500	2.25000	1.12500	−2.04167	−2.04167	0.70834
2	1.15057	2.79600	1.39835	−2.15911	−2.92955	0.88968
3	1.18189	3.17442	1.49157	−2.25220	−3.30192	1.00153
4	1.20011	3.37940	1.53371	−2.30863	−3.47115	1.06840
5	1.20975	3.49342	1.55575	−2.34384	−3.55892	1.11010
λ	A_2	B_2	C_2	D_2	E_2	F_2
1	1.12500	2.25000	1.12500	−2.04167	−2.04167	0.70834
2	1.13309	2.06325	0.78911	−2.02992	−1.42616	0.62377
3	1.13762	2.04430	0.68017	−2.04007	−1.29319	0.61164
4	1.13968	2.04486	0.64402	−2.04620	−1.25386	0.60930
5	1.14074	2.04744	0.62940	−2.04968	−1.23862	0.60884

Table 1 (B) The values of coefficients for U_2, V_2 in (4.8a, b) when ν=0.30

λ	A_1	B_1	C_1	D_1	E_1	F_1
1	1.11667	2.23333	1.11667	−2.01667	−2.01667	0.68333
2	1.12585	2.78453	1.39402	−2.08489	−2.92156	0.81479
3	1.13279	3.17261	1.48901	−2.10788	−3.30292	0.87458
4	1.12114	3.38259	1.53210	−2.08840	−3.47524	0.89767
5	1.09958	3.49866	1.55467	−2.05252	−3.56378	0.90489
λ	A_2	B_2	C_2	D_2	E_2	F_2
1	1.11667	2.23333	1.11667	−2.01667	−2.01667	0.68333
2	1.13057	2.05329	0.77981	−2.02120	−1.40966	0.61730
3	1.13643	2.03825	0.67519	−2.03573	−1.28684	0.60949
4	1.13899	2.04100	0.64216	−2.04365	−1.25235	0.60856
5	1.14030	2.04481	0.62322	−2.04803	−1.23919	0.60862

Table1(c) The values of coefficients for U_2, V_2 in (4.8a, b) when $\nu=0.35$

λ	A_1	B_1	C_1	D_1	E_1	F_1
1	1.10833	2.21667	1.10833	−1.99167	−1.99167	0.65833
2	1.10100	2.77331	1.38966	−2.00963	−2.91403	0.73711
3	1.08266	3.17142	1.48643	−1.95854	−3.30471	0.73948
4	1.03923	3.38564	1.53048	−1.85643	−3.48026	0.71275
5	0.98369	3.50486	1.55358	−1.74105	−3.56961	0.67981
λ	A_2	B_2	C_2	D_2	E_2	F_2
1	1.10833	2.21667	1.10833	−1.99167	−1.99167	0.65833
2	1.12804	2.04328	0.77066	−2.01244	−1.39371	0.61122
3	1.13524	2.03215	0.67057	−2.03137	−1.28142	0.60775
4	1.13830	2.03710	0.64080	−2.04110	−1.25190	0.60818
5	1.13985	2.04217	0.62959	−2.04636	−1.24083	0.60871

It is shown in the above tables that in the case of circular plate, $\lambda^2=1/\lambda^2=1$, and the values of all the coefficients are equal to the corresponding values given in (4.23), (4.25) and (4,27).

In the above, we have the first approximation solution of the large deflection problem of elliptical plate.

V. The Second Approximation Solution

The substitution of the first approximation solution (4.4), (4.8a, b) into the equation of the second approximation (3.5a) gives the differential equation for the determination of $W_3(\xi, \eta)$ and α_3 :

$$
\begin{aligned}
&\frac{\partial^4 W_3}{\partial\xi^4}+2\lambda^2\frac{\partial^4 W_3}{\partial\xi^2\partial\eta^2}+\lambda^4\frac{\partial^4 W_3}{\partial\eta^4}-8(3+2\lambda^2+3\lambda^4)\alpha_3\\
&\quad=48(1-3\xi^2-\eta^2)\{(7A_1+\nu\lambda^2C_2)\xi^6+[5(A_1+B_1)+3\nu\lambda^2(B_2+C_2)]\xi^4\eta^2\\
&\qquad+[3(B_1+C_1)+5\lambda^2\nu(A_2+B_2)]\eta^4\xi^2+(C_1+7\nu\lambda^2A_2)\eta^6+[-5(A_1-D_1)\\
&\qquad-\nu\lambda^2(C_2-E_2)]\xi^4+[-3(B_1-D_1-E_1)-3\nu\lambda^2(B_2-D_2-E_2)]\xi^2\eta^2\\
&\qquad-[(C_1-E_1)+5\nu\lambda^2(A_2-D_2)]\eta^4-[3(D_1-F_1)+\nu\lambda^2(E_2-F_2)]\xi^2\\
&\qquad-[(E_1-F_1)+3\nu\lambda^2(D_2-F_2)]\eta^2-[F_1+\nu\lambda^2F_2]\}\\
&\qquad+48\lambda^2(1-\xi^2-3\eta^2)\{(7\nu A_1+\lambda^2C_2)\xi^6+[5\nu(A_1+B_1)+3\lambda^2(B_2+C_2)]\xi^4\eta^2\\
&\qquad+[3\nu(B_1+C_1)+5\lambda^2(A_2+B_2)]\eta^4\xi^2+[\nu C_1+7\lambda^2A_2]\eta^6+[-5\nu(A_1-D_1)\\
&\qquad-\lambda^2(C_2-E_2)]\xi^4\\
&\qquad+[-3\nu(B_1-D_1-E_1)-3\lambda^2(B_2-D_2-E_2)]\xi^2\eta^2+[-\nu(C_1-E_1)-5\lambda^2(A_2-D_2)]\eta^4\\
&\qquad+[-3\nu(D_1-F_1)-\lambda^2(E_2-F_2)]\xi^2+[-\nu(E_1-F_1)-3\lambda^2(D_2-F_2)]\eta^2\\
&\qquad-[\nu F_1+\lambda^2F_2]\}\\
&\qquad-192(1-\nu)\lambda^2\xi\eta\{(A_1+B_1+3C_2)\eta\xi^5+2(B_1+C_1+B_2+C_2)\xi^3\eta^3\\
&\qquad+(3C_1+A_2+B_2)\eta^5\xi+[-B_1+E_1+D_1-2(C_2-E_2)]\eta\xi^3+[-2(C_1-E_1)\\
&\qquad-(B_2-E_2-D_2)]\xi\eta^3+(-E_1+F_1-E_2+F_2)\eta\xi\}\\
&\qquad+384(1-\xi^2-\eta^2)^2\{(3+\nu\lambda^2)\xi^4+[1+2(2+\nu)\lambda^2+\lambda^4]\xi^2\eta^2\\
&\qquad+(\nu+3\lambda^2)\lambda^2\eta^4-(1+\nu\lambda^2)\xi^2-(\nu+\lambda^2)\lambda^2\eta^2\}
\end{aligned}
\tag{5.1}
$$

At the same time, the solution of (5.1) may be of written as:

$$W_3(\xi,\ \eta)=(1-\xi^2-\eta^2)^2\{G\xi^8+H\xi^6\eta^2+I\xi^4\eta^4+J\xi^2\eta^6+K\eta^8+M\xi^6$$
$$+N\xi^4\eta^2+L\xi^2\eta^4+P\eta^6+Q\xi^4+R\xi^2\eta^2+S\eta^4+T\xi^2+X\eta^2\} \tag{5.2}$$

in which $G,\ H,\ I,\ J,\ K,\ M,\ N,\ L,\ P,\ Q,\ R,\ S,\ T,\ X$ are 14 constants to be determined. It can be see that $W_3(\xi,\eta)$ in (5.2) satisfies the boundary conditions (3.6). Substituting (5.2) into the left-hand sides of (5.1) gives:

$$\frac{\partial^4 W_3}{\partial\xi^4}+2\lambda^2\frac{\partial^4 W_3}{\partial\xi^2\partial\eta^2}+\lambda^4\frac{\partial^4 W_3}{\partial\eta^4}$$

$$\begin{aligned}
&=24\xi^8\{(495+30\lambda^2+\lambda^4)G+(15+2\lambda^2)\lambda^2H+\lambda^4I\}+24\xi^6\eta^2\{(420+56\lambda^2)G\\
&+(210+112\lambda^2+15\lambda^4)H+(56+30\lambda^2)\lambda^2I+15\lambda^4J\}+120\xi^4\eta^4\{14G+(28+15\lambda^2)H\\
&+(14+30\lambda^2+14\lambda^4)I+(15+28\lambda^2)\lambda^2J+14\lambda^4K\}+24\xi^2\eta^6\{15H+(30+56\lambda^2)I\\
&+(15+112\lambda^2+210\lambda^4)J+(56+420\lambda^2)\lambda^2K\}+24\eta^8\{I+(2+15\lambda^2)J\\
&+(1+30\lambda^2+495\lambda^4)K\}+8\xi^6\{-(1260+56\lambda^2)G-6\lambda^4I-(56+6\lambda^2)\lambda^2H\\
&+(630+56\lambda^2+3\lambda^4)M+(28+6\lambda^2)\lambda^2N+3\lambda^4L\}+120\xi^4\eta^2\{-28G-(28+12\lambda^2)H\\
&-(12+6\lambda^2)\lambda^2I-6\lambda^4J+6(1+\lambda^2)\lambda^2L+(28+6\lambda^2)M+(14+12\lambda^2+3\lambda^4)N\\
&+3\lambda^4P\}+120\xi^2\eta^4\{-6H-(6+12\lambda^2)I-(12+28\lambda^2)\lambda^2J-28\lambda^4K+3M\\
&+6(1+\lambda^2)N+(3+12\lambda^2+14\lambda^4)L+(6\lambda^2+28\lambda^4)P\}+8\eta^6\{-6I-(6+56\lambda^2)J\\
&+3N+(6+28\lambda^2)L-(56+1260\lambda^2)\lambda^2K+(3+56\lambda^2+630\lambda^4)P\}\\
&+24\xi^4\{70G+5\lambda^2H+\lambda^4I-(140+10\lambda^2)M-(10+2\lambda^2)\lambda^2N-2\lambda^4L\\
&+(70+10\lambda^2+\lambda^4)Q+(5+2\lambda^2)\lambda^2R+\lambda^4S\}\\
&+72\xi^2\eta^2\{5H+4\lambda^2I+5\lambda^4J-(8+10\lambda^2)\lambda^2L-10M\\
&-(10+8\lambda^2)N+(10+4\lambda^2)Q-10\lambda^4P+(5+8\lambda^2+5\lambda^4)R+(4+10\lambda^2)\lambda^2S\}\\
&+24\eta^4\{I+5\lambda^2J+70\lambda^4K-(2+10\lambda^2)L-2N-(10+140\lambda^2)\lambda^2P+Q+(2+5\lambda^2)R\\
&+(1+10\lambda^2+70\lambda^4)S\}+24\xi^2\{\lambda^4L+15M+2\lambda^2N-(30+4\lambda^2)Q-2(2+\lambda^2)\lambda^2R\\
&-2\lambda^4S+(15+4\lambda^2+\lambda^4)T+2(1+\lambda^2)\lambda^2X\}+24\eta^2\{2\lambda^2L+N+15\lambda^4P-2Q\\
&-2(1+2\lambda^2)R-(4+30\lambda^2)\lambda^2S+2(1+\lambda^2)T+(1+4\lambda^2+15\lambda^4)X\}\\
&+8\{3Q+\lambda^2R+3\lambda^4S-2(3+\lambda^2)T-2(1+3\lambda^2)\lambda^2X\}
\end{aligned} \tag{5.3}$$

Comparing the coefficients of various terms in (5.1) with (5.3) gives all the equations for the determination of various undetermined constants. The coefficients of ξ^8, $\xi^6\eta^2$, $\xi^4\eta^4$, $\xi^2\eta^6$, η^8 give 5 equations for the determination of 5 constants of $G,\ H,\ I,\ J,\ K$.

$$\left.\begin{aligned}
&(495+30\lambda^2+\lambda^4)G+(15+2\lambda^2)\lambda^2H+\lambda^4I=f_1\\
&(420+56\lambda^2)G+(210+112\lambda^2+15\lambda^4)H+(56+30\lambda^2)\lambda^2I+15\lambda^4J=f_2\\
&70G+(140+75\lambda^2)H+(70+150\lambda^2+70\lambda^4)I+(75+140\lambda^2)\lambda^2J+70\lambda^4K=f_3\\
&15H+(30+56\lambda^2)I+(15+112\lambda^2+210\lambda^4)J+(56+420\lambda^2)\lambda^2K=f_4\\
&I+(2+15\lambda^2)J+(1+30\lambda^2+495\lambda^4)K=f_5
\end{aligned}\right\} \tag{5.4}$$

in which f_i ($i=1,\ 2,\ 3,\ 4,\ 5$) are the following known functions of λ^2 and ν , which are obtained from the right-hand side of (5.1).

$$f_1=-6(7A_1+\nu\lambda^2C_2)-2\lambda^2(7\nu A_1+\lambda^2C_2)+16(3+\nu\lambda^2) \tag{5.5a}$$

$$f_2=-2(7A_1+\nu\lambda^2C_2)-6[5(A_1+B_1)+3\nu\lambda^2(C_2+B_2)]-2\lambda^2[5\nu(A_1+B_1)+3\lambda^2(B_2+C_2)]$$
$$-6\lambda^2(7\nu A_1+\lambda^2C_2)-8(1-\nu)\lambda^2(A_1+B_1+3C_1)+32(3+\nu\lambda^2)$$
$$+16[1+2(2+\nu)\lambda^2+\lambda^4] \tag{5.5b}$$
$$f_3=-6[3(B_1+C_1)+5\nu\lambda^2(A_2+B_2)]-2[5(A_1+B_1)+3\nu\lambda^2(B_2+C_2)]-6\lambda^2[5\nu(A_1+B_1)$$
$$+3\lambda^2(C_2+B_2)]+2\lambda^2[3\nu(B_1+C_1)+5\lambda^2(A_2+B_2)]-16(1-\nu)\lambda^2[(B_1+C_1)$$
$$+(B_2+C_2)]+16(3+\nu\lambda^2)+32[1+2(2+\nu)\lambda^2+\lambda^4]+16(\nu+3\lambda^2)\lambda^2 \tag{5.5c}$$
$$f_4=-6(C_1+7\nu\lambda^2A_2)-2[3(B_1+C_1)+5\nu\lambda^2(A_2+B_2)]-6\lambda^2[3\nu(B_1+C_1)$$
$$+5\lambda^2(A_2+B_2)]-2\lambda^2(\nu C_1+7\lambda^2A_2)-8(1-\nu)\lambda^2[3C_1+A_2+B_2]+32(\nu+3\lambda^2)\lambda^2$$
$$+16[1+2(2+\nu)\lambda^2+\lambda^4] \tag{5.5d}$$
$$f_5=-2(C_1+7\nu\lambda^2A_2)-6\lambda^2(\nu C_1+7\lambda^2A_2)+16(\nu+3\lambda^2)\lambda^2 \tag{5.5e}$$

It should be noted that A_1, B_1, C_1, A_2, B_2, C_2 are given in (4.16a, b), and therefore, f_i ($i=1, 2, 3, 4, 5$) in (5.5) are known. Thus, (5.4) are 5 linear equations for the determination of 5 unknowns G, H, I, J, K. This can be written in matrix form as follows:

$$\boldsymbol{\mu}_1\mathbf{X}_1=\boldsymbol{\Omega}_1 \tag{5.6}$$

in which

$$\mathbf{X}_1=(G\ H\ I\ J\ K)^T,\qquad \boldsymbol{\Omega}_1=(f_1\ f_2\ f_3\ f_4\ f_5)^T \tag{5.7a, b}$$

$$\boldsymbol{\mu}_1=\begin{bmatrix} 495+30\lambda^2+\lambda^4 & (15+2\lambda^2)\lambda^2 & \lambda^4 & 0 & 0 \\ 420+56\lambda^2 & 210+112\lambda^2+15\lambda^4 & (56+30\lambda^2)\lambda^2 & 15\lambda^4 & 0 \\ 70 & 140+75\lambda^2 & 70+150\lambda^2+70\lambda^4 & (75+140\lambda^2)\lambda^2 & 70\lambda^4 \\ 0 & 15 & 30+56\lambda^2 & 15+112\lambda^2+210\lambda^4 & (56+420\lambda^2)\lambda^2 \\ 0 & 0 & 1 & 2+15\lambda^2 & 1+30\lambda^2+495\lambda^4 \end{bmatrix} \tag{5.7c}$$

The solution of (5.6) can be written as follows:

$$\mathbf{X}_1=\boldsymbol{\mu}_1^{-1}\boldsymbol{\Omega}_1 \tag{5.8}$$

In (5.1) and (5.3), the coefficients of terms ξ^6, $\xi^4\eta^2$, $\xi^2\eta^4$, η^6 are given.

$$(1260+56\lambda^2)G+(56+6\lambda^2)\lambda^2H+6\lambda^4I-(630+56\lambda^2+3\lambda^4)M-(28+6\lambda^2)\lambda^2N$$
$$-3\lambda^4L=f_6 \tag{5.9a}$$
$$28G+(28+12\lambda^2)H+(12+6\lambda^2)\lambda^2I+6\lambda^4J-(28+6\lambda^2)M-(14+12\lambda^2+3\lambda^4)N$$
$$-6\lambda^2(1+\lambda^2)L-3\lambda^4P=f_7 \tag{5.9b}$$
$$6H+(6+12\lambda^2)I+(12+28\lambda^2)\lambda^2J+28\lambda^4K-3M-6(1+\lambda^2)N$$
$$-(3+12\lambda^2+14\lambda^4)L-\lambda^2(6+28\lambda^2)P=f_8 \tag{5.9c}$$
$$6I+(6+56\lambda^2)J-3N-(6+28\lambda^2)L+(56+1260\lambda^2)\lambda^2K-(3+56\lambda^2+630\lambda^4)P=f_9 \tag{5.9d}$$

In which, G, H, I, J, K are known as shown in (5.8), and therefore, (5.9a, b, c, d) is a system of 4 equations for the determination of 4 unknown L, M, N, P. In matrix form, it can be written as:

$$\boldsymbol{\mu}_2\mathbf{X}_1-\boldsymbol{\theta}_2\mathbf{X}_2=-\frac{2}{5}\boldsymbol{\Omega}_2 \tag{5.10}$$

in which X_1 is shown as in (5.7a); X_2, μ_2, θ_2, Ω_2 are

$$X_2=\{M\ N\ L\ P\}^T,\qquad \Omega_2=\{15f_6\ f_7\ f_8\ 15f_9\}^T \tag{5.11a, b}$$

$$\mu_2=\begin{bmatrix} 1260+56\lambda^2 & (56+6\lambda^2)\lambda^2 & 6\lambda^4 & 0 & 0 \\ 28 & 28+12\lambda^2 & (12+6\lambda^2)\lambda^2 & 6\lambda^4 & 0 \\ 0 & 6 & 6+12\lambda^2 & (12+28\lambda^2)\lambda^2 & 28\lambda^4 \\ 0 & 0 & 6 & 6+56\lambda^2 & (56+1260\lambda^2)\lambda^2 \end{bmatrix} \tag{5.11c}$$

$$\theta_2=\begin{bmatrix} 630+56\lambda^2+3\lambda^4 & (28+6\lambda^2)\lambda^2 & 3\lambda^4 & 0 \\ 28+6\lambda^2 & 14+12\lambda^2+3\lambda^4 & 6\lambda^2(1+\lambda^2) & 3\lambda^4 \\ 3 & 6(1+\lambda^2) & 3+12\lambda^2+14\lambda^4 & (6+28\lambda^2)\lambda^2 \\ 0 & 3 & 6+28\lambda^2 & 3+56\lambda^2+630\lambda^4 \end{bmatrix} \tag{5.11d}$$

The solution of (5.10) is

$$X_2=\theta_2^{-1}\{\mu_2X_1+2\Omega_2/5\} \tag{5.12}$$

The values of f_6, f_7, f_8, f_9 in (5.11b) are respectively:

$$\begin{aligned} f_6=&(7A_1+\nu\lambda^2C_2)+3[5(A_1-D_1)+\nu\lambda^2(C_2-E_2)]+(7\nu A_1+\lambda^2C_2)\lambda^2 \\ &+[5\nu(A_1-D_1)\lambda^2+(C_2-E_2)\lambda^4]-8[2(3+\nu\lambda^2)+(1+\nu\lambda^2)] \end{aligned} \tag{5.13a}$$

$$\begin{aligned} f_7=&[5(A_1+B_1)+3\nu\lambda^2(B_2+C_2)]+9[B_1-D_1-E_1+\nu\lambda^2(B_2-D_2-E_2)] \\ &+[5(A_1-D_1)+\nu\lambda^2(C_2-E_2)]+\lambda^2[5\nu(A_1+B_1)+3\lambda^2(B_2+C_2)] \\ &+3\lambda^2[\nu(B_1-D_1-E_1)+\lambda^2(B_2-D_2-E_2)]+3\lambda^2[5\nu(A_1-D_1)+\lambda^2(C_2-E_2)] \\ &+4(1-\nu)\lambda^2(B_1-E_1-D_1+2C_2-2E_2)-16[4+(4+3\nu)\lambda^2+\lambda^4]-8(2+3\nu\lambda^2+\lambda^4) \end{aligned} \tag{5.13b}$$

$$\begin{aligned} f_8=&[3(B_1+C_1)+5\nu\lambda^2(A_2+B_2)]+3[B_1-D_1-E_1+\nu\lambda^2(B_2-D_2-E_2)] \\ &+3[C_1-E_1+5\nu\lambda^2(A_2-D_2)]+\lambda^2[3\nu(B_1+C_1)+5\lambda^2(A_2+B_2)]+\lambda^2[\nu(C_1-E_1) \\ &+5\lambda^2(A_2-D_2)]+3\lambda^2[3\nu(B_1-D_1-E_1)+3\lambda^2(B_2-D_2-E_2)]+4(1-\nu)\lambda^2 \\ &\cdot[2(C_1-E_1)+B_2-E_2-D_2]-16[1+2(2+\nu)\lambda^2+\lambda^4]-16\lambda^2(\nu+3\lambda^2) \\ &-16\lambda^2(\nu+\lambda^2)-8(1+\nu\lambda^2) \end{aligned} \tag{5.13c}$$

$$\begin{aligned} f_9=&(C_1+7\nu\lambda^2A_2)+[C_1-E_1+5\nu\lambda^2(A_2-D_2)]+(\nu C_1+7\lambda^2A_2)\lambda^2+3[\nu(C_1-E_1) \\ &+5\lambda^2(A_2-D_2)]\lambda^2-16(\nu+3\lambda^2)\lambda^2-8(\nu+\lambda^2)\lambda^2 \end{aligned} \tag{5.13d}$$

(5.12) gives X_2 or gives M, N, L, P.

Comparing the coefficients of terms ξ^4, $\xi^2\eta^2$, η^4 in (5.1) with (5.3) gives:

$$\begin{aligned} &70G+5\lambda^2H+\lambda^4I-(140+10\lambda^2)M-(10+2\lambda^2)\lambda^2N-2\lambda^4L+(70+10\lambda^2+\lambda^4)Q \\ &+(5+2\lambda^2)\lambda^2R+\lambda^4S=f_{10} \end{aligned} \tag{5.14a}$$

$$\begin{aligned} &15H+12\lambda^2I+15\lambda^4J-3(8+10\lambda^2)\lambda^2L-30M-3(10+8\lambda^2)N+3(10+4\lambda^2)Q \\ &-30\lambda^4P+3(5+8\lambda^2+5\lambda^4)R+3(4+10\lambda^2)\lambda^2S=f_{11} \end{aligned} \tag{5.14b}$$

$$\begin{aligned} &I+5\lambda^2J+70\lambda^4K-(2+10\lambda^2)L-2N-10(1+14\lambda^2)\lambda^2P+Q \\ &+(2+5\lambda^2)R+(1+10\lambda^2+70\lambda^4)S=f_{12} \end{aligned} \tag{5.14c}$$

in which f_{10}, f_{11}, f_{12} are respectively:

$$f_{10}=-10(1+\nu)\lambda^2(A_1-D_1)-2(\nu+\lambda^2)\lambda^2(C_2-E_2)+6(3+\nu\lambda^2)(D_1-F_1)$$

$$+2(3\nu+\lambda^2)\lambda^2(E_2-F_2)+16(5+3\nu\lambda^2) \tag{5.15a}$$

$$f_{11}=2[3+4\lambda^2-3\nu\lambda^2](E_1-F_1)+2[4-3\nu+3\lambda^2]\lambda^2(E_2-F_2)+6(3\nu+\lambda^2)\lambda^2(D_2-F_2)$$
$$+6(1+3\nu\lambda^2)(D_1-F_1)-6(1+\nu\lambda^2)(B_1-D_1-E_1)-6(\nu+\lambda^2)\lambda^2(B_2-D_2-E_2)$$
$$+16[3+2(2+3\nu)\lambda^2+3\lambda^4] \tag{5.15b}$$

$$f_{12}=-2(1+\nu\lambda^2)(C_1-E_1)-10(\nu+\lambda^2)\lambda^2(A_2-D_2)+2(1+3\nu\lambda^2)(E_1-F_1)$$
$$+6(\nu+3\lambda^2)\lambda^2(D_2-F_2)+16(3\nu+5\lambda^2)\lambda^2 \tag{5.15c}$$

(5.14) may be written in matrix form:

$$\mu_3 X_1+\theta_3 X_2+\omega_3 X_3=\Omega_3 \tag{5.16}$$

in which X_1, X_2 are shown as in (5.7a) and (5.11a), while μ_3, θ_3, ω_3, Ω_3, X_3 are respectively:

$$X_3=\begin{Bmatrix} Q \\ R \\ S \end{Bmatrix},\quad \Omega_3=\begin{Bmatrix} f_{10} \\ f_{11} \\ f_{12} \end{Bmatrix},\quad \mu_3=\begin{bmatrix} 70 & 5\lambda^2 & \lambda^4 & 0 & 0 \\ 0 & 15 & 12\lambda^2 & 15\lambda^4 & 0 \\ 0 & 0 & 1 & 5\lambda^2 & 70\lambda^4 \end{bmatrix} \tag{5.17a,b,c}$$

$$\theta_3=\begin{bmatrix} -(140+10\lambda^2) & -(10+2\lambda^2)\lambda^2 & -2\lambda^4 & 0 \\ -30 & -3(10+8\lambda^2) & -3(8+10\lambda^2)\lambda^2 & -30\lambda^4 \\ 0 & -2 & -(2+10\lambda^2) & (10+140\lambda^2)\lambda^2 \end{bmatrix} \tag{5.17d}$$

$$\omega_3=\begin{bmatrix} 70+10\lambda^2+\lambda^4 & (5+2\lambda^2)\lambda^2 & \lambda^4 \\ 3(10+4\lambda^2) & 3(5+8\lambda^2+5\lambda^4) & 3(4+10\lambda^2)\lambda^2 \\ 1 & 2+5\lambda^2 & 1+10\lambda^2+70\lambda^4 \end{bmatrix} \tag{5.17e}$$

The solution of (5.16) gives Q, R, S, i.e.

$$X_3=\omega_3^{-1}\{\Omega_3/3-\mu_3 X_1-\theta_3 X_2\} \tag{5.18}$$

Comparing of the coefficients of various terms ξ^2, η^2 in (5.1) with (5.3) gives:

$$\lambda^4 L+15M+2\lambda^2 N-(30+4\lambda^2)Q-(4+2\lambda^2)\lambda^2 R-2\lambda^4 S+(15+4\lambda^2+\lambda^4)T$$
$$+2(1+\lambda^2)\lambda^2 X=f_{13} \tag{5.19a}$$

$$2\lambda^2 L+N+15\lambda^4 P-2Q-2(1-2\lambda^2)R-(4+30\lambda^2)\lambda^2 S+2(1+\lambda^2)T$$
$$+(1+4\lambda^2+15\lambda^4)X=f_{14} \tag{5.19b}$$

in which f_{13}, f_{14} respectively are:

$$f_{13}=-6(1+\nu\lambda^2)(D_1-F_1)-2\lambda^2(\nu+\lambda^2)(E_2-F_2)+2(3+\nu\lambda^2)F_1+2(3\nu+\lambda^2)\lambda^2 F_2$$
$$-16(1+\nu\lambda^2) \tag{5.20a}$$

$$f_{14}=-2(1+\nu\lambda^2)(E_1-F_1)-6\lambda^2(\nu+\lambda^2)(D_2-F_2)+2(1+3\nu\lambda^2)F_1+2(\nu+3\lambda^2)\lambda^2 F_2$$
$$-16(\nu+\lambda^2)\lambda^2 \tag{5.20b}$$

(5.19a, b) may be written in matrix form as follows:

$$\theta_4 X_2+\omega_4 X_3+\pi_4 X_4=\Omega_4 \tag{5.21}$$

in which X_2, X_3 are shown as in (5.11a), (5.17a), while X_4, Ω_4, θ_4, ω_4, π_4 are respectively:

$$X_4=\begin{Bmatrix} T \\ X \end{Bmatrix},\quad \Omega_4=\begin{Bmatrix} f_{13} \\ f_{14} \end{Bmatrix},\quad \theta_4=\begin{bmatrix} 15 & 2\lambda^2 & \lambda^4 & 0 \\ 0 & 1 & 2\lambda^2 & 15\lambda^4 \end{bmatrix} \tag{5.22a,b,c}$$

$$\boldsymbol{\omega}_4=\begin{bmatrix} (30+4\lambda^2) & -(4+2\lambda^2)\lambda^2 & -2\lambda^4 \\ -2 & -2(1+2\lambda^2) & -(4+30\lambda^2)\lambda^2 \end{bmatrix} \tag{5.22d}$$

$$\boldsymbol{\pi}_4=\begin{bmatrix} 15+4\lambda^2+\lambda^4 & 2(1+\lambda^2)\lambda^2 \\ 2(1+\lambda^2) & 1+4\lambda^2+15\lambda^4 \end{bmatrix} \tag{5.22e}$$

the solution of (5.2) gives T, X. They are:

$$\mathbf{X}_4=\boldsymbol{\pi}_4^{-1}(\boldsymbol{\Omega}_4-\boldsymbol{\theta}_4\mathbf{X}_2-\boldsymbol{\omega}_4\mathbf{X}_3) \tag{5.23}$$

At last, comparing of constant terms in (5.1) with that of (5.3) gives:

$$(3+2\lambda^2+3\lambda^4)a_3=3Q+\lambda^2R+3\lambda^4S-2(3+\lambda^2)T-2(1+3\lambda^2)\lambda^2X+6(1+\nu\lambda^2)F_1 \\ +6\lambda^2(\nu+\lambda^2)F_2 \tag{5.24}$$

in which Q, R, S, T, X are shown respectively from $\mathbf{X}_3$, $\mathbf{X}_4$ or from (5.18) and (5.23), F_1, F_2 are shown respectively from $\boldsymbol{\Psi}_1$, $\boldsymbol{\Psi}_2$ in (4.20).

Based upon the results of calculation in the 2nd order perturbation, this unknown coefficients in (5.2) for $W_3(\xi, \eta)$ are completely determined. The unknown constant a_3 is also obtain. The values of these coefficients for different λ with ν =0.25, 0.30, 0.35 are given in Table 2 (A, B, C).

Table 2 (A) The coefficients in (5.2) and a_3 in (5.24) with ν=0.25

λ	G	H	I	J	K
1	-0.52083×10^{-2}	-0.20833×10^{-1}	-0.31250×10^{-1}	-0.20833×10^{-1}	-0.52083×10^{-2}
2	-0.39984×10^{-1}	-0.52988×10^{-1}	-0.19562×10^{-1}	-0.51014×10^{-2}	-0.51610×10^{-3}
3	-0.11305	-0.87392×10^{-1}	-0.79593×10^{-2}	-0.16386×10^{-2}	-0.12148×10^{-3}
4	-0.21263	-0.11740	-0.30407×10^{-2}	-0.64242×10^{-3}	-0.41687×10^{-4}
5	-0.31431	-0.14190	-0.10610×10^{-2}	-0.29418×10^{-3}	-0.17816×10^{-4}

λ	M	N	L	P	Q
1	0.36458×10^{-1}	0.10938	0.10938	0.36458×10^{-1}	-0.11719
2	0.24550	0.21857	0.36759×10^{-1}	0.41229×10^{-2}	-0.64741
3	0.64191	0.33111	0.11728×10^{-1}	0.10113×10^{-2}	-1.45782
4	1.11020	0.42083	0.39038×10^{-2}	0.35335×10^{-3}	-2.22038
5	1.52727	0.48999	0.11689×10^{-2}	0.15263×10^{-3}	-2.79620

λ	R	S	T	X	a_3
1	-0.23438	-0.11719	0.33681	0.33681	0.53733
2	-0.33633	-0.15188×10^{-1}	0.93368	0.21976	0.58232
3	-0.44380	-0.36244×10^{-2}	1.55911	0.21914	0.63610
4	-0.52117	-0.10883×10^{-2}	1.99581	0.22522	0.66999
5	-0.57538	-0.33718×10^{-3}	2.27442	0.2301.	0.68934

Table 2 (B) The coefficients in (5.2) and a_3 in (5.24) with ν=0.30

λ	G	H	I	J	K
1	-0.50556×10^{-2}	-0.20222×10^{-1}	-0.30333×10^{-1}	-0.20222×10^{-1}	-0.50556×10^{-2}
2	-0.38868×10^{-1}	-0.51510×10^{-1}	-0.18997×10^{-1}	-0.49270×10^{-2}	-0.50150×10^{-3}
3	-0.11061	-0.85186×10^{-1}	-0.78521×10^{-2}	-0.15566×10^{-2}	-0.11820×10^{-3}
4	-0.20942	-0.11476	-0.31870×10^{-2}	-0.59740×10^{-3}	-0.40580×10^{-4}
5	-0.31099	-0.13913	-0.13148×10^{-2}	-0.26760×10^{-3}	-0.17343×10^{-4}

Continue **Table 2 (B)**

λ	M	N	L	P	Q
1	0.35389×10^{-1}	0.10617	0.10617	0.35389×10^{-1}	-0.11375
2	0.23927	0.21258	0.35785×10^{-1}	0.39870×10^{-2}	-0.63502
3	0.63083	0.32328	0.11751×10^{-1}	0.96847×10^{-3}	-1.44202
4	1.09773	0.41254	0.42840×10^{-2}	0.33463×10^{-3}	-2.20583
5	1.51572	0.48211	0.18476×10^{-2}	0.14302×10^{-3}	-2.78420

λ	R	S	T	X	α_3
1	-0.22750	-0.11375	0.33656	0.33656	0.54564
2	-0.32806	-0.14814×10^{-1}	0.93108	0.21977	0.58713
3	-0.43563	-0.36637×10^{-2}	1.55569	0.21806	0.63872
4	-0.51406	-0.12047×10^{-2}	1.99280	0.22512	0.67127
5	-0.56946	-0.45863×10^{-3}	2.27203	0.23001	0.69043

Table 2(c) The coefficients in (5.2) and α_3 in (5.24) with $\nu=0.35$

λ	G	H	I	J	K
1	-0.48750×10^{-2}	-0.19500×10^{-1}	-0.29250×10^{-1}	-0.19500×10^{-1}	-0.48750×10^{-2}
2	-0.37537×10^{-1}	-0.49757×10^{-1}	-0.18321×10^{-1}	-0.47271×10^{-2}	-0.48411×10^{-3}
3	-0.10754	-0.82559×10^{-1}	-0.76774×10^{-2}	-0.14679×10^{-2}	-0.11426×10^{-3}
4	-0.20503	-0.11159	-0.32840×10^{-2}	-0.55036×10^{-3}	-0.39247×10^{-4}
5	-0.30603	-0.13578	-0.15304×10^{-2}	-0.24024×10^{-3}	-0.16773×10^{-4}

λ	M	N	L	P	Q
1	0.34125×10^{-1}	0.10238	0.10238	0.34125×10^{-1}	-0.10969
2	0.23168	0.20550	0.34586×10^{-1}	0.38301×10^{-2}	-0.61898
3	0.61615	0.31393	0.11656×10^{-1}	0.92098×10^{-3}	-1.41849
4	1.07935	0.40246	0.45874×10^{-2}	0.31437×10^{-3}	-2.18091
5	1.49685	0.47233	0.20724×10^{-2}	0.13275×10^{-3}	-2.76095

λ	R	S	T	X	α_3
1	-0.21938	-0.10969	0.33525	0.33525	0.55294
2	-0.31820	-0.14344×10^{-1}	0.92455	0.21931	0.59077
3	-0.42563	-0.36685×10^{-2}	1.54615	0.21860	0.64018
4	-0.50509	-0.13036×10^{-2}	1.98308	0.22469	0.67188
5	-0.56178	-0.56969×10^{-3}	2.26325	0.22964	0.69068

VI. Relation between Central Deflection and Uniform Loading

From (3.2d), we may write the relation between the central deflection and uniform loading in the second approximation as follows:

$$3Q/2(3+2\lambda^2+3\lambda^4)=\alpha_1 W_m+\alpha_3 W_m^3 \tag{6.1}$$

in which Q is dimensionless uniform loading. W_m is dimensionless central deflection (see (2.4)). α_1 and α_3 are respectively shown in (4.4) and (5.24).

$$\alpha_1=1 \tag{6.2}$$

α_3 is the function of λ^2 and ν. The various values of λ^2 and ν, α_3 are shown in Table 2 (A, B, C).

When $\lambda^2=1/\lambda^2=1$, we have:

$$3/2(3+2\lambda^2+3\lambda^4)=3/16 \qquad (6.3)$$

(6.1) is of the same form as shown for circular plate, see[1] (1948). When $\lambda^2=1, \nu=0.30$ we have:

$$a_3=0.54564 \qquad (6.4)$$

which is the same as shown in [1] (1948).

Based upon this relation given in (6.1), we can constructs the central deflection curves against uniform loading.

Fig. 2 shows the central deflection W_m curves against the intensity of uniform loading for various λ^2 when $\nu=0.30$.

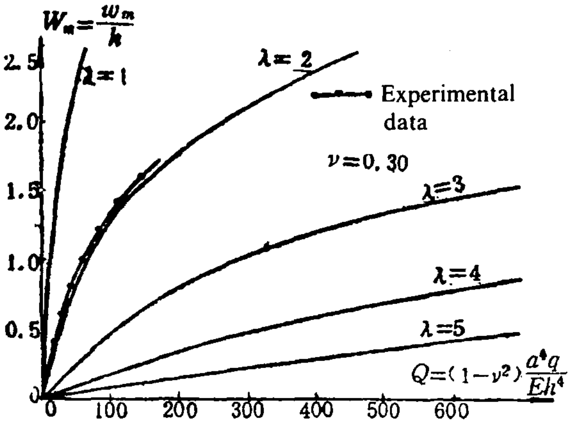

Fig. 2 Central deflection against intensity of uniform loading for various λ^2, when $\nu=0.30$

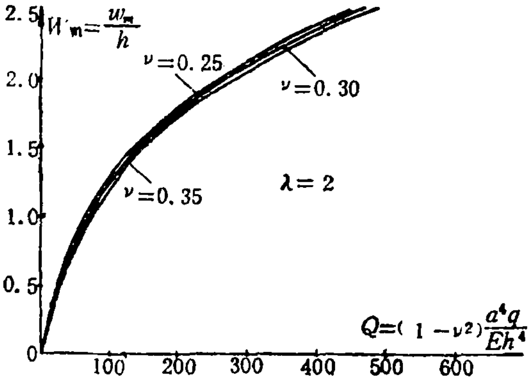

Fig. 3 Central deflection curves against intensity of uniform loading for various ν values, when $\lambda=2$.

Fig. 3 shows the central deflection curves against intensity of uniform loading for various ν values when $\lambda=2$.

In Fig.2, the experimental point "." are given by Nash and Cooley for $\lambda^2=4$(1959)[3].

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