

ON THE INEFFICIENCY OF THE QUASI-GRADIENT SCREENING ALGORITHM

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Abstract

In this paper, the well-known quasi-gradient screening algorithm on optimal sequencing cascaded development of water energy resources will be introduced. Then we will given a contraexample in practice and prove the inefficiency of the algorithm in theory.

Key words quasi-gradient, algorithm, optimal sequencing, cascaded hydroelectric station

I. Introduction

The problem on optimal sequencing cascaded development of water energy resources is called capacity expansion abroad and ordinal problem in operations research. This problem may be described as follows: Given that n stations in a river are to be constructed one after another, how do you determine the sequence of these stations to get the maximum total profits? Many well-known techniques, such as dynamic programming, integer programming, linear programming, simulation model, random research technique, etc., or their improved forms have been introduced to deal with this problem by many authors such as W. S. Butcher, Y. Y. Haimes, T. L. Morin, A. M. O. Esogbue, D. Erlenkotter, D. T. O'laughaire, W. S. Nainnis, D. P. Loucks, J. Kuiper, L. Ortolano, M. R. Rao, P. J. Doulliez, J. A. Bondy, U. S. R. Murty, R. A. Rhode, H. Luss, Feng shang-you, Zhang Yi, Li Mi-an, etc. (cf. [1], [2]). Unfortunately, since the calculation times of all the techniques except our results (see [3], [4]) and the methods in [5] grow exponentially with the number of stations, the techniques are not very successful, especially when the number of stations is large for example, larger than 7).

The authors in [5] introduced the quasi-gradient screening algorithm, and they proved that the algorithm is P algorithm (the calculation times to determine the optimal sequence are polynomials of the number of stations) when the quasi-gradient isn't equal to zero. Unlike most researchers who try to study the problem by the improvement of the well-known operations research techniques, the authors in [5] became fully aware of the limitations of all the well-known techniques in studying the capacity expansion. So they tried to find an effective algorithm on the optimal sequencing of cascaded hydroelectric stations. The authors in [5] introduced a new concept named "quasi-gradient", and discussed the properties of the concept. Furthermore, based on the properties of the

quasi-gradient, they attained several theorems and the so-called quasi-gradient screening algorithm to determine the optimal sequence. The authors in [5] did a large amount of hard and meticulous work, and their results were highly evaluated, too (see [6]). But we have to indicate that the algorithm is wrong both in theory and in practice. We'll give a contraexample in practice and prove the inefficiency of the algorithm in theory in section III.

II. Brief Introduction to Quasi-Gradient Screening Algorithm

In the convenience of narration, we first quote some definitions and notations introduced in [5].

Definition 1 If two sequences are different only in the order of a pair of neighbour elements (or stations), the two sequences are called neighbour sequences.

Definition 2 A project is a station or an imaginary station whose function is equivalent to that of the combination of several stations.

Definition 3 The difference of the objective values of two neighbour sequences is called the quasi-gradient caused by the exchange of two neighbour elements, we denote the quasi-gradient as $QG(P_1 \bar{A} \bar{B} P_2)$,

$$QG(P_1 \bar{A} \bar{B} P_2) = NB(P_1 A B P_2) - NB(P_1 B A P_2)$$

where $NB(P_1 A B P_2)$ and $NB(P_1 B A P_2)$ are the objective values (current value of the net profits) corresponding to the two sequences $P_1 A B P_2$ and $P_1 B A P_2$, A and B are two stations, and P_1 and P_2 are subsequences of some stations or projects.

Conclusion 1 If $QG(P_1 \bar{A} \bar{B} P_2) > 0$, $P_1 B A P_2$ isn't the optimal sequencing.

The main points of the quasi-gradient screening algorithm are as follows:

Step 1 Select two stations or projects A , B , calculate the quasi-gradient caused by the exchange of A and B by means of

$$QG(\bar{A} \bar{B}) = QG(P_1 \bar{A} \bar{B}) = NB(P_1 A B) - NB(P_1 B A) \quad (2.1)$$

If $QG(\bar{A} \bar{B}) > 0$, then we are sure that $B A$ isn't but $A B$ may be the optimal decision.

Step 2 Classify the probable optimal decisions screened out by step 1 in their first element. Select two decisions $\bar{A} B$ and $\bar{A} C$ from any classification, and calculate $QG(\bar{A} B \bar{A} C)$ by means of

$$QG(\bar{A} B \bar{A} C) = QG(\bar{A} \bar{B} \bar{C}) \quad (2.2)$$

If $QG(\bar{A} B \bar{A} C) > 0$, then we are sure that $\bar{A} C$ isn't but $\bar{A} B$ may be the optimal decision. Imitate the method above, after we have screened many times, a probable optimal decision in any classification is screened out.

Step 3 Select two decisions $\bar{A} B$ and $\bar{C} D$ from the decision screened out by step 2 and calculate $QG(\bar{A} B \bar{C} D)$ by means of

$$QG(\bar{A} B \bar{C} D) = QG(\bar{A} \bar{B} \bar{D}), \quad QG(\bar{A} B \bar{C} D) = QG(\bar{A} B \bar{C}) \quad (2.3)$$

If $QG(\bar{A} B \bar{C} D) > 0$, then we're sure that $\bar{C} D$ isn't but $\bar{A} B$ may be the optimal decision. Repeat it in this way, an optimal decision is screened out. Namely, the subsequence of two elements in the optimal sequencing is selected.

Step 4 Now, for the remaining $(K-2)$ stations, repeat step 1 until the remaining number is 0 or 1.

III. Contraexample and Theoretic Analysis

Given that four stations are to be constructed in a river one after another, and the conditions of these stations are as in Tab. 1. And we suppose that the discount rate is 0.05. Let's determine the sequence of construction to make the current total net profits maximal.

Table 1

| Ordinal number of station | 1 | 2 | 3 | 4 |
|--|--------------------|-----------------------|-----------------------|-----------------------|
| Characteristics | Regulating station | Run-off-river station | Run-off-river station | Run-off-river station |
| Net profits when it works independently | 740 | 800 | 850 | 1000 |
| Net profits raised by the construction of the regulating station | — | 300 | 100 | 50 |
| Period of construction | 8.5 | 8 | 8 | 8 |

According to the quasi-gradient screening algorithm the optimal sequencing is 4 3 1 2; its current net profits are 1560.83. Nevertheless, according to our study (cf. [3], [4]), the optimal sequencing is 4 2 1 3; its current net profits are 1567.72. Namely, the sequencing gotten by quasi-gradient screening algorithm may not be the optimal one. The reason why the case occurs is that step 2 and step 3 of the algorithm are wrong. Because the definition and conclusion of quasi-gradient (i.e. def. 3 and conclusion 1) are defined only when elements are stations but not projects. If the quasi-gradient caused by the exchange of two projects is defined according to (2.1), (2.2) and (2.3), conclusion 1 isn't true. Now let's see the detailed reason.

First, for the convenience of narration, we suppose that there are only four stations A, B, C, D . Second, Let V_X be the net profits of station X when it works independently, and $V_{XY} = V_{YX}$ the profits raised by the co-work of station X and Y , $Z = 1/(1+r)$ where r is the discount rate. And let m_1, m_2, m_3, m_4 be the periods of construction of stations A, B, C and D respectively.

Because $QG(\overline{AB} \overline{AC}) = QG(A \overline{B} \overline{C}) = NB(ABCD) - NB(ACBD)$,

If $QG(\overline{AB} \overline{AC}) > 0$, i.e. $NB(ABCD) - NB(ACBD) > 0$, we're only sure that $ACBD$ isn't the optimal sequencing, but we are not sure that AC isn't the subsequencing of the first two elements of the optimal sequencing.

Based on the formula of profits we have

$$\begin{aligned}
 NB(ABCD) &= V_A Z^{m_1} + (V_B + V_{AB}) Z^{m_1+m_2} + (V_C + V_{AC} + V_{BC}) Z^{m_1+m_2+m_3} \\
 &\quad + (V_D + V_{AD} + V_{BD} + V_{CD}) Z^{m_1+m_2+m_3+m_4} \\
 NB(ACBD) &= V_A Z^{m_1} + (V_C + V_{AC}) Z^{m_1+m_3} + (V_B + V_{AB} + V_{BC}) Z^{m_1+m_2+m_3} \\
 &\quad + (V_D + V_{AD} + V_{BD} + V_{CD}) Z^{m_1+m_2+m_3+m_4}
 \end{aligned}$$

from which we have

$$QG(\overline{AB} \overline{AC}) = Z^{m_1} ((V_B + V_{AB}) Z^{m_2} (1 - Z^{m_3}) - (V_C + V_{AC}) Z^{m_3} (1 - Z^{m_2})) \quad (3.1)$$

According to the quasi-gradient screening algorithm, if $(3.1) > 0$ we're sure that AC isn't the optimal subsequencing of the first two elements of the optimal sequencing. Nevertheless, we have

$$\begin{aligned}
 NB(ACDB) &= V_A Z^{m_1} + (V_C + V_{AC}) Z^{m_1+m_3} + (V_D + V_{AD} + V_{CD}) Z^{m_1+m_3+m_4} \\
 &\quad + (V_B + V_{AB} + V_{CB} + V_{DB}) Z^{m_1+m_2+m_3+m_4} \\
 NB(ABDC) &= V_A Z^{m_1} + (V_B + V_{AB}) Z^{m_1+m_2} + (V_D + V_{AD} + V_{BD}) Z^{m_1+m_2+m_4} \\
 &\quad + (V_C + V_{AC} + V_{BC} + V_{DC}) Z^{m_1+m_2+m_3+m_4}
 \end{aligned}$$

Therefore,

$$\begin{aligned} \text{NB}(ACDB) - \text{NB}(ABCD) = & Z^{m_1}((V_O + \bar{V}_{AO})Z^{m_2}(1 - Z^{m_2}) + (V_D \\ & + \bar{V}_{AD})Z^{m_3+m_4}(1 - Z^{m_2}) - (V_B + \bar{V}_{AB})Z^{m_2}(1 - Z^{m_3+m_4}) \\ & + \bar{V}_{OD}Z^{m_3+m_4}(1 - Z^{m_2}) - \bar{V}_{CB}Z^{m_2+m_3}(1 - Z^{m_4})) \end{aligned} \quad (3.2)$$

$$\begin{aligned} \text{NB}(ACDB) - \text{NB}(ABDC) = & Z^{m_1}((V_O + \bar{V}_{AO})Z^{m_2}(1 - Z^{m_2+m_4}) \\ & - (V_B + \bar{V}_{AB})Z^{m_2}(1 - Z^{m_3+m_4}) + \bar{V}_{OD}Z^{m_3+m_4}(1 - Z^{m_2}) \\ & - \bar{V}_{DB}Z^{m_2+m_4}(1 - Z^{m_3})) \end{aligned} \quad (3.3)$$

That (3.1), (3.2), (3.3) may all be greater than 0, and (3.2), (3.3) > 0 tells us that AC may be the optimal subsequencing of the first two elements of the optimal sequencing but AB isn't. Namely, the true optimal decision may be screened out wrongly and another decision may be regarded as the optimal one by the quasi-gradient screening algorithm. In fact, it's very easy to make (3.1), (3.2), (3.3) > 0 . For the convenience of narration, we suppose $m_1 = m_2 = m_3 = m_4 = M$ and let $R = Z^M$, then (3.1), (3.2), (3.3) may be rewritten as follows:

$$R^2(1-R)((V_B + \bar{V}_{AB}) - (V_O + \bar{V}_{AO})) \quad (3.4)$$

$$\begin{aligned} R^2(1-R)((V_D + \bar{V}_{AD} + \bar{V}_{CD} - \bar{V}_{CB} - (V_B + \bar{V}_{AB}))R \\ - ((V_B + \bar{V}_{AB}) - (V_O + \bar{V}_{AO}))) \end{aligned} \quad (3.5)$$

$$R^2(1-R)((\bar{V}_{CD} - \bar{V}_{DB})R - ((V_B + \bar{V}_{AB}) - (V_O + \bar{V}_{AO}))(1+R)) \quad (3.6)$$

Obviously, if we give V_D , \bar{V}_{CD} , \bar{V}_{DB} proper values, the three expressions above can all be greater than 0.

To sum up, the so-called quasi-gradient screening algorithm can't assure that its optimal sequencing is really optimal. So the algorithm is ineffective both in theory and in practice. But we should mention that the algorithm is true, if all the stations are run-off-river stations. To get the detailed understanding of this point, please see [3]. We regret that the authors of [5] didn't consider it from another view. Therefore they didn't point out that their results may be efficient in this special case.

IV. Concluding Remarks

The essence of the optimal sequencing of cascaded hydroelectric stations is to prove whether this problem is P problem or not. Any satisfactory methods to deal with this problem haven't been attained until now. Many scientists are devoting themselves to this difficult problem. We hope that with the continuous effort of colleagues all over the world this problem will be solved satisfactorily soon.

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