Applied Mathematics and Mechanics
(English Edition, Vol. 17, No. 11, Nov. 1996)

Published by SU,
Shanghai, China

A NON-INCREMENTAL TIME-SPACE ALGORITHM FOR NUMERICAL SIMULATION OF FORMING PROCESS

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(Received April 22, 1996)

Abstract

A non-incremental time-space algorithm is proposed for numerical analysis of forming process with the inclusion of geometrical, material, contact-frictional non-linearities. Unlike the widely used Newton-Raphson counterpart, the present scheme features an iterative solution procedure on entire time and space domain. Validity and feasibility of the present scheme are further justified by the numerical investigation herewith presented.

Key words forming process, numerical simulation, non-incremental algorithm, time-space function

I. Introduction

The significant role played by the development in and investigations on numerical simulation procedure in forming process has increasingly more widely accepted by manufacture industries because of its remarkable contribution in shortening the period required for determination of the forming technology and in bringing down the expenses. However, the multiplicity of non-linearities in high degrees involved in the forming process immensely hinders it, theoretically and numerically, from being extended to practical applications. In dealing with time factors in problems regarding processes, incremental algorithms are usually employed. The fact that small increments are necessarily required in such practices unavoidably induces convergence difficulties in addition to disadvantageously low efficiency[1, 2, 3]. This is especially worse for engineering applications where immense amount of computational effort is usually required.

Contrast to the conventional approaches, the non-incremental algorithm features a concept that the whole loading process is considered as a unique increment. In a series of publications[4, 5, 6], P. Ladeveze initiated the so-called "large time increment method" for small deformation elasto-plastic problems, which was subsequently applied, in success, to several research projects[7, 8, 9, 10, 11].

The present study is mainly concerned with the implementation of, and development in, the concepts of the non-incremental algorithm[12], with specific emphasis on the description of

* Project supported by the National Natural Science Foundation of China

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time-process and its solution procedures in connection with numerical simulations for finite-deformation forming process. As long as iterative manipulations are conducted on the whole time process, amount of global solution efforts is drastically brought down, while the expenses required by global solution efforts are in power series relations with the number of degrees-of-freedom involved in a discrete model[13, 14]. Solution variables, in the present study, are composed of the products of time functions with space functions[13].

II. Statement of the Problem and the Presentation of the Method

Let t be the time: $t\in[0,T]$, V the initial configuration of the workpiece at $t=0$. The boundary of the workpiece is divided into three disjoint parts: S_1 is the part of boundary where displacement $\bar{U}$ is prescribed, S_2 is the part of boundary where load $\bar{F}$ is prescribed, and S_3 is the part of boundary where possibly contact friction with the tools takes place[15, 16, 17] The current position (at time t) of the initial material coordinate M is $M_t=M+u(M, t)$, with u as the single-valued continuous displacement field. V_t is the configuration at time t, resulted from V via displacement u, while the boundary is transformed correspondingly to its current component parts (Fig. 1).

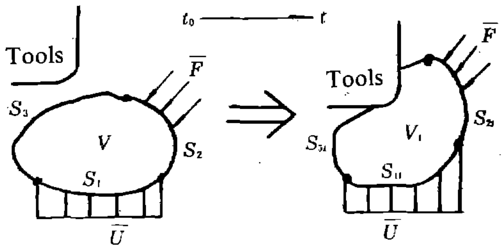

Fig. 1 Mechanical model

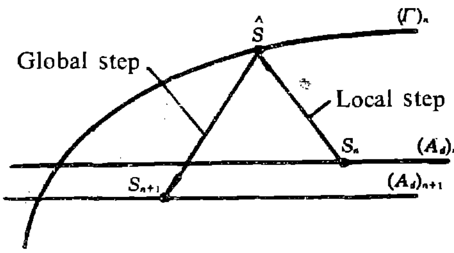

Fig. 2 The iteration procedure

Introduce new variables: load q and displacement v on S_3[18, 19]. Denote p as the first Piola-Kirchhoff stress tensor and F the deformation gradient tensor, i.e. $F=I+du/dM$, with I as identity tensor, such that the space of variables is $S=(F, v, p, q)$.

To discribe the virtual strain-rate field and the virtual stress-rate field, define the statically and kinematically admissible to zero set S_A and K_A:

$$U=\{u\,|\,u=u(M,t),\quad u|_{S_1}=0,\quad u|_{t=0}=0\} \tag{2.1}$$

$$K_A=\left\{(\dot{F},\dot{v})\,|\,\exists\, u\in U,\ \dot{F}=\frac{d\dot{u}}{dM},\quad \dot{v}=\dot{u}\Big|_{S_3}\right\} \tag{2.2}$$

$$S_A=\left\{(\dot{p},\dot{q})\,|\,\forall\,(F^*,v^*)\in K_A,\ \int_0^T\left[\int_V Tr(\dot{p}^T F^*)dV+\int_{S_3}\dot{q}^T v^* dS\right]dt=0\right\} \tag{2.3}$$

wherein superindex T implies transpose.

Ignoring body force exertions, the mechanics problem is written as:

- $F=I+\dfrac{du}{dM},\quad u\Big|_{S_1}=\bar{u},\quad v=u\Big|_{S_3}$ $\qquad$ (2.4)

- (p,q) satisfies

$$\int_V Tr[p^T F^*]\,dv+\int_{S_3} q^T v^* d\check{S}=\int_{S_2}\bar{F}u^* dS \qquad (\forall u^*\in K_A) \tag{2.5}$$

- $\dot{F}=A(P_\tau,\ \tau\leqslant t)$ (material constutive law[20, 21]) $\qquad$ (2.6)

• $R(q,\dot{v})$ (contact friction law[22,23]) (2.7)

Define $A_d=\{S|S$ satisfies (2.4)(2.5)$\}$ (2.8)

$\Gamma=\{S|S$ satisfies (2.6)(2.7)$\}$ (2.9)

where A_d is a global linear set, satisfying the statically and kinematically admissible condition, and Γ comprises all the local non-linear problems.

Let each iteration be split into two steps, the local step and the global step, over $V\times[0,T]$ and $S_3\times[0,T]$. In the first local (local) step, starting with a statically and kinematically admissible solution $S_n\in A_d$, solution $\hat{S}\in\Gamma$ is determined. In the second (global) step, from $\hat{S}$, a new solution $S_{n+1}\in A_d$ is determined, which is superior to S_n. Keeping going ahead, the procedure is to successively converge to the vicinity of exact solution $S_e=A_d\cap\Gamma$ (Fig. 2).

Search directions for these two steps are:

$$\text{Local Step}\quad \left.\begin{aligned}&\hat{\dot{p}}-\dot{p}_n=-K_0(\hat{\dot{F}}-\dot{F}_n) && \in V\times[0,T]\\ &\hat{\dot{q}}-\dot{q}_n=-k(\hat{\dot{v}}-\dot{v}_n) && \in S_3\times[0,T]\end{aligned}\right\} \tag{2.10}$$

$$\text{Global Step}\quad \left.\begin{aligned}&\hat{\dot{p}}-\dot{p}_{n+1}=K_0(\hat{\dot{F}}-\dot{F}_{n+1}) && \in V\times[0,T]\\ &\hat{\dot{q}}-\dot{q}_{n+1}=k(\hat{\dot{v}}-\dot{v}_{n+1}) && \in S_3\times[0,T]\end{aligned}\right\} \tag{2.11}$$

where K_0 is the linear elastic material constant and k the surface stiffness parameter[12]. The n-th iterative modifiers are:

$$\Delta S_n=(\Delta\dot{p}_n,\ \Delta\dot{F}_n,\ \Delta\dot{q}_n,\ \Delta\dot{v}_n) \tag{2.12}$$

Taking account of equations (2.10), (2.11), (2.4) and (2.5), the global variational statement is:

Find ΔS_n which satisfies

$$\left.\begin{aligned}&\Delta\dot{p}_n-K_0\Delta\dot{F}_n=2K_0(\dot{F}_n-\hat{\dot{F}}) && \in V\times[0,T]\\ &\Delta\dot{q}_n-k\Delta\dot{v}_n=2k(\dot{v}_n-\hat{\dot{v}}) && \in S_3\times[0,T]\end{aligned}\right\} \tag{2.13}$$

$$\left.\begin{aligned}&(\exists\, u_n\in U),\quad \Delta\dot{F}_n=\frac{d\Delta\dot{u}_n}{dM},\quad \Delta\dot{v}_n=\Delta\dot{u}_n\Big|_{S_3}\\ &(u^*\in K_A),\quad \int_0^T\left\{\int_V Tr[\Delta\dot{p}_n^T F^*]\,dV+\int_{S_3}\Delta\dot{q}_n^T v^* dS\right\}dt=0\end{aligned}\right\} \tag{2.14}$$

Having the static variables $\Delta\dot{p}_n$ and $\Delta\dot{q}_n$ eliminated, the solution unknowns $\Delta\dot{F}_n$ and $\Delta\dot{v}_n$ are available as solutions of displacement variational problem:

Find $(\Delta\dot{F}_n,\ \Delta\dot{v}_n)$, which satisfies $(\Delta\dot{F}_n,\ \Delta\dot{v}_n)\in K_A$, $\forall(F^*,v^*)\in K_A$

$$\begin{aligned}&\int_V\int_0^T Tr[F^{*T}K_0\Delta\dot{F}_n]\,dtdV+\int_{S_3}\int_0^T v^{*T}k\Delta\dot{v}_n dtdS\\ &=2\int_V\int_0^T Tr[F^{*T}(\dot{p}_n-\hat{\dot{p}})]\,dtdV+2\int_{S_3}\int_0^T v^{*T}(\dot{q}_n-\hat{\dot{q}})\,dtdS\end{aligned} \tag{2.15}$$

Alternatively, when kinematical variables $(\Delta\dot{F}_n,\ \Delta\dot{v}_n)$ be eliminated, $(\Delta\dot{p}_n,\ \Delta\dot{q}_n)$ will be available as solution of stress variational problem, i. e.

Find $(\Delta\dot{p}_n,\ \Delta\dot{q}_n)$, which satisfies $(\Delta\dot{p}_n,\ \Delta\dot{q}_n)\in S_A$, $\forall(p^*,q^*)\in S_A$

$$\int_V\int_0^T Tr[p^{*T}K_0^{-1}\Delta\dot{p}_n]dtdV+\int_{S_3}\int_0^T q^{*T}k^{-1}\Delta\dot{q}_n dtdS$$
$$=2\int_V\int_0^T Tr[p^{*T}(\dot{F}_n-\hat{\dot{F}})]dtdV+2\int_{S_3}\int_0^T q^{*T}(\dot{v}_n-\hat{\dot{v}})dtdS \qquad (2.16)$$

Finally, solutions to be obtained by use of the non-incremental method are represented as

$$\left.\begin{aligned} m\in[1,3],\quad &\Delta\dot{u}_n=\sum_{i=1}^m g_i(t)\omega_i(M),\quad \Delta\dot{F}_n=\sum_{i=1}^m g_i(t)\alpha_i(M)\\ &\Delta\dot{v}_n=\sum_{i=1}^m g_i(t)\gamma_i(M),\quad \Delta\dot{p}_n=\sum_{i=1}^m h_i(t)\beta_i(M)\\ &\Delta\dot{q}_n=\sum_{i=1}^m h_i(t)\,\delta_i(M),\quad \alpha_i=\frac{d\omega_i}{dM},\ \gamma_i=\omega_i\Big|_{S_3}\end{aligned}\right\} \qquad (2.17)$$

III. Local Step——Pre-Determination of the Time Function

In local step, increments of kinematic variables are

$$\Delta\hat{\dot{F}}(M,\ t)=\hat{\dot{F}}-\dot{F}_n\cong\sum_{i=1}^m \hat{g}_i(t)\hat{\alpha}_i(M),\quad \Delta\hat{\dot{v}}(M,t)=\hat{\dot{v}}-\dot{v}_n\cong\sum_{i=1}^m\hat{g}_i(t)\hat{\gamma}_i(M) \qquad (3.1)$$

In order that definite solutions can be obtained, $\hat{g}_i(t)$ is normalized via $\int_0^T\hat{g}_i(t)\hat{g}_i(t)dt=1$.

1. First order approximation ($m=1$)

To ensure that the right-hand-sides converge to the left-hand-sides of Eq. (3.1), the difference $\Delta\hat{\dot{F}}-\hat{g}(t)\hat{\alpha}(M)$ and the difference $\Delta\hat{\dot{v}}-\hat{g}(t)\hat{\gamma}(M)$ are required to be minimal. Accordingly,

$$L=L(\hat{g},\hat{\alpha},\ \hat{\gamma})=\int_0^T\left\|\Delta\hat{\dot{F}}-\hat{g}(t)\hat{\alpha}\,(M)\right\|_{(1)}^2dt+\int_0^T\left\|\Delta\hat{\dot{v}}-\hat{g}(t)\hat{\gamma}\,(M)\right\|_{(2)}^2dt \qquad (3.2)$$

should assume extrema. Note that in the above equation,

$$\left\|X\right\|_{(1)}^2=\int_V X^TK_0XdV,\quad \left\|X\right\|_{(2)}^2=\int_V X^TkXdV \qquad (3.3)$$

As such, the problem turns to: find $\hat{g},\ \hat{\alpha},\ \hat{\gamma}$,

$$\forall\delta\alpha,\ \forall\delta\gamma,\ \forall\delta g,\ \delta L(\hat{g},\ \hat{\alpha},\ \hat{\gamma})=0 \qquad (3.4)$$

In fact, by use of the normalization condition, one obtains the Euler equations

$$\hat{\alpha}=\int_0^T\hat{g}(t)\Delta\hat{\dot{F}}dt,\quad \hat{\gamma}=\int_0^T\hat{g}(t)\Delta\hat{\dot{v}}dt \qquad (3.5)$$

Substituting (3.5) into (3.2) will have the L-minima problem converted to

$$\underset{\displaystyle\hookrightarrow\hat{g},\ \int_0^T\hat{g}(t)\hat{g}(t)dt=1}{\mathrm{Sup}\,M_1}=\left\|\int_0^T\hat{g}(t)\Delta\hat{\dot{F}}dt\right\|_{(1)}^2+\left\|\int_0^T\hat{g}(t)\Delta\hat{\dot{v}}dt\right\|_{(2)}^2 \qquad (3.6)$$

Moreover, introduction of the normalization condition, via the use of Lagrangian multiplitier λ, converts the problem to

$$\delta\left\{\left\|\int_0^T \hat{g}(t)\Delta\hat{\boldsymbol{F}}dt\right\|^2_{(1)}+\left\|\int_0^T \hat{g}(t)\Delta\hat{v}dt\right\|^2_{(2)}+\lambda\left[\int_0^T \hat{g}\ \hat{g}\ dt-1\right]\right\}=0 \qquad (3.7)$$

which will be further posed as an eigen-pair problem regarding operator A:

$$\hat{g}\Rightarrow A(\hat{g})=\int_V Tr[\Delta\hat{\boldsymbol{F}}^T K_0\left(\int_0^T \hat{g}(t)\Delta\hat{\boldsymbol{F}}dt\right)]dV+\int_{S_3}\Delta\hat{v}^T k\left(\int_0^T \hat{g}(t)\Delta\hat{v}dt\right)dV \qquad (3.8)$$

where A is a symmetric positive-definite operator. Maxima of M_1 is available via the eigen vector $\hat{g}_1$ associated with the maximum eigen value λ_1.

2. Second-order approximation

On the basis of the first-order approximations $\hat{g}_1\hat{\alpha}_1$ and $\hat{g}_1\hat{\nu}_1$, second-order modification terms $\hat{g}_2\hat{\alpha}_2$ and $\hat{g}_2\hat{\nu}_2$ are added.

Define

$$\hat{X}=\Delta\hat{\boldsymbol{F}}-\hat{g}_1(t)\hat{\alpha}_1(M),\quad \hat{x}=\Delta\hat{v}-\hat{g}_1(t)\hat{\gamma}_1(M) \qquad (3.9)$$

Following the same procedure as what has been done with the first-order approximation terms, except the replacement of $\Delta\hat{\boldsymbol{F}}$ and $\Delta\hat{v}$ with $\hat{X}$ and $\hat{x}$ respectively, one has

$$\sup_{\hat{g},\ \int_0^T \hat{g}(t)\hat{g}(t)dt=1} M_2=\left\|\int_0^T \hat{g}(t)\Delta\hat{\boldsymbol{F}}dt\right\|^2_{(1)}+\left\|\int_0^T \hat{g}(t)\Delta\hat{v}\,dt\right\|^2_{(2)} \qquad (3.10)$$

Taking into account the two components of $\hat{g}$, one in the known direction $\hat{g}_1$ to which the other component $\bar{g}$ is orthogonal, it is apparent that

$$\hat{g}(t)=c\hat{g}_1+\bar{g}(t)\quad\text{where}\quad \int_0^T \hat{g}_1(t)\bar{g}(t)dt=0 \qquad (3.11)$$

and the normalization condition

$$c^2+\int_0^T \bar{g}(t)\bar{g}(t)dt=1$$

On such basis, the use of equation (3.5) gives rise to

$$\int_0^T \hat{g}\hat{X}\,dt=\int_0^T \bar{g}\Delta\hat{\boldsymbol{F}}dt\quad\text{and}\quad\int_0^T \hat{g}\ \hat{x}dt=\int_0^T \bar{g}\Delta\hat{v}dt \qquad (3.12)$$

which yield $M_2(\hat{g}_2)=M_1(\bar{g}_2)$. Further, the introduction of normalization condition via the use of Lagrangian multiplier λ will have the problem under consideration converted to the solution of the following variational problem:

$$\delta_{c\in R,\ \bar{g}(t)\perp\hat{g}_1(t),\ \lambda\in R}\left\{M_1(\bar{g})+\lambda\left(c^2+\int_0^T \bar{g}\bar{g}dt-1\right)\right\}=0 \qquad (3.13)$$

the Euler equation thereof comprises

$$A(\bar{g})+2\lambda\bar{g}=0,\ c^2+\int_0^T \bar{g}(t)\bar{g}(t)dt-1=0,\ 2c\lambda=0 \tag{3.14}$$

from which arises $c=0$. Consequently, the solution to this system is orthogonal to $\hat{g}_1$. Actually, this is an eigenvalue problem; the eigen-value is λ_2, to which related the eigen-vector $\hat{g}_2$.

IV. Solution of the Space Function

Based on (2.17), the procedure of solving the variational problem (2.15) can be stated as: Find $\omega_j\in K_A,\ j\in[1,m]$, such that

$$\forall i\in[1,m],\ \forall\omega_i^*\in K_A,\ \sum_{j=1}^m\int_V Tr\left[\alpha_i^{*T}\int_0^T \hat{g}_i(t)K_0\hat{g}_j(t)\,dt\alpha_j\right]dV$$

$$+\sum_{j=1}^m\int_{S_3}\gamma_i^{*T}\int_0^T\hat{g}_i(t)k\hat{g}_j(t)dt\gamma_j dS=-2\int_V Tr[\alpha_i^{*T}\Phi_i]\,dV-2\int_{S_3}\gamma_i^{*T}\varphi_i dS \tag{4.1}$$

where

$$\Phi_i(M)=\int_0^T \hat{g}_i(t)(\hat{\boldsymbol{p}}-\dot{p}_n)dt,\quad \varphi_i(M)=\int_0^T\hat{g}_i(t)(\hat{q}-\dot{q}_n)dt \tag{4.2}$$

By use of the ortho-normal condition, the following m independent variational equations emerge, namely

$$\forall i\in[1,m],\ \forall\omega_i^*\in K_A$$

$$\int_V Tr[\alpha_i^{*T}K_0\alpha_i]\,dV+\int_{S_3}\gamma_i^{*T}k\gamma_i dS=-2\int_V Tr[\alpha_i^{*T}\Phi_i]\,dV-2\int_{S_3}\gamma_i^{*T}\varphi_i dS \tag{4.3}$$

Substituting the pre-determined value $\hat{g}_i(t)$, which are known from the local steps, into (4.3), the kinematically admissible to zero variables ω_i, α_i and γ_i can be resulted, which it is desirable to know in expressing $\Delta\hat{\boldsymbol{F}}_n$ and $\Delta\hat{v}_n$.

Alternatively, Eq. (4.3) can be written as

$$\forall i\in[1,m],\ \forall(\alpha_i^*,\gamma_i^*)\in K_A$$

$$\int_V Tr\{\alpha_i^{*T}[K_0\alpha_i+2\Phi_i]\}dV+\int_{S_3}\gamma_i^{*T}[k\gamma_i+2\varphi_i]dS=0 \tag{4.4}$$

In the above equation, square bracket denotes the kinematically admissible to zero variables.

Hence, the (β_i,δ_i), $i\in[1,m]$ variables, totaling m sets, are available via

$$\beta_i(M)=K_0\alpha_i+2\Phi_i,\quad \delta_i(M)=k\gamma_i+2\varphi_i \tag{4.5}$$

Note that $\Delta\hat{\boldsymbol{p}}_n$ and $\Delta\dot{q}_n$ are expressed in terms of β_i and δ_i respectively [see Eq. (2.17)].

V. Determination of Time Variables

On the basis of (2.16) and (2.17), with (β_i,δ_i) already known, the time function $h_i(t)$ is available via (2.16), on the following defined numerical space. In fact, letting H be a real space defined on [0, T], the present problem can be written as

$$(h_1,\cdots,h_m)\in H^m,\quad\forall(h_1^*,\cdots,h_m^*)\in H^m$$

$$\sum_{j=1}^{m}\int_0^T h_i^*(t)H_{ij}h_j(t)dt=\int_0^T h_i^*(t)b_i(t)dt \tag{5.1}$$

where

$$H_{ij}=\int_V Tr[\beta_i^T K_0^{-1}\beta_j]dV+\int_{S_3}\delta_i^T k^{-1}\delta_j dS \tag{5.2}$$

is an $m\times m$ symmetric constant matrix, and

$$b_i(t)=2\int_V Tr[\beta_i^T K_0^{-1}(\dot{\hat{p}}-\dot{p}_n)\,dV+2\int_{S_3}\delta_i^T k^{-1}(\dot{\hat{q}}-\dot{q}_n)dS \tag{5.3}$$

Namely, at each discrete time point, an m-dimensional linear problem should be solved, i. e.

$$\sum_{j=1}^{m}H_{ij}h_j(t)=b_i(t) \tag{5.4}$$

In like manner, with (α_i,γ_i) known, the time function $g_i(t)$ can be obtained via displacement variational solution (2.15). In fact,

$$\sum_{j=1}^{m}G_{ij}g_j(t)=c_i(t) \tag{5.5}$$

where

$$G_{ij}=\int_V Tr[\alpha_i^T K_0\alpha_j]dV+\int_{S_2}\gamma_i^T k\gamma_j dS \tag{5.6}$$

$$c_i(t)=2\int_V Tr[\alpha_i^T K_0(\dot{\hat{F}}-\dot{F}_n)]\,dV+2\int_{S_3}\gamma_i^T k(\dot{\hat{v}}-\dot{v}_n)dS \tag{5.7}$$

Starting with the $g_i(t)$ thus obtained, repeat the solution procedures 4 and 5, results with improved accuracy can be obtained. However, for iterative approach, strickly accurate calculations in a single step is practically not necessary.

VI. Numerical Examples

The present algorithm is implemented by use of the finite element analysis software OPTIFORM[24] in conjunction with the pre-processing package AUTMESH and a post-processing package specifically developed by means of a graphic tool.

Table 1 Material harding property

ε_p plastic strain	0.00	0.10	0.30	0.40
σ stress (MPa)	200.0	400.0	500.0	600.0

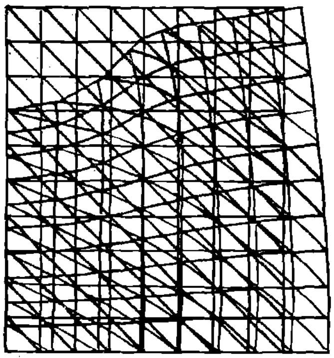

Fig. 3 Deformation at indentation depth equal to 15mm

To examine the validity and feasibility of the present algorithm, a plane strain elasto-plastic problem is analyzed, in which a rigid circular cylinder, 25mm in radius, is indented into an 80mm×50mm rectangular structure, with elastic modulus $E=2\times10^5$MPa, Poisson's ratio $\nu=0.3$ and piece-wise linear harding material (Table 1), obeying Prandlt-Reuss elasto-plastic constitive, Von Mises criteria and Coulomb contact friction law. Fig. 3 gives the configuration and deformed mesh at indentation depth equal to 15mm. For symmetry, computational results of half structure are provided.

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